



# Gain Fluctuations of Cryogenic Amplifiers

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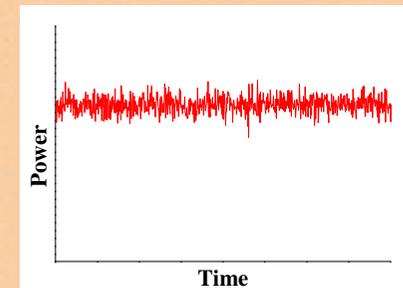
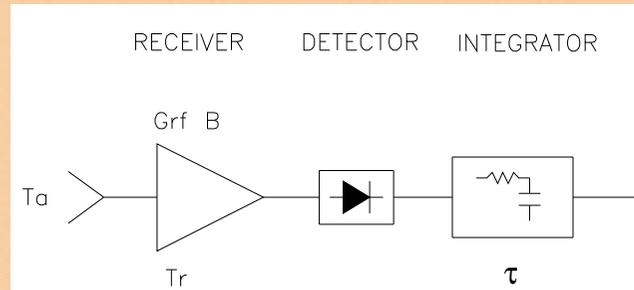
- Introduction
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# Radiometer Sensitivity

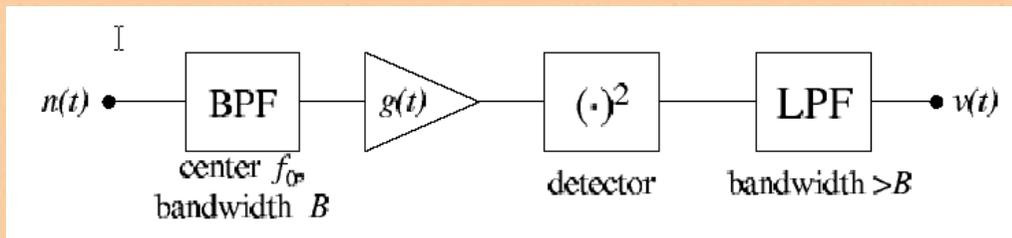
## ■ Radiometer

$$P_{out} = K \cdot G_{rf} \cdot B \cdot (T_a + T_r)$$

$$\frac{\Delta P_{out}}{P_{out}} = \frac{k}{\sqrt{B\tau}} + \frac{\Delta G_{rf}}{G_{rf}}$$



## ■ Radiometer model



## ■ Radiometer sensitivity (total power radiometer)

$$\left( \frac{S_v(f)}{V_0} \right)^2 = \delta(f) + \frac{2}{B} + \left( \frac{S_g(f)}{G_0} \right)^2, \quad |f| \ll B$$

# Gain Stability Characterization

## ■ Allan Variance of Normalized Gain (unitless):

- Time domain
- Astronomer's choice

$$\sigma^2(\tau) = \frac{1}{2} \left\langle \left( \overline{G}(t + \tau) - \overline{G}(t) \right)^2 \right\rangle$$

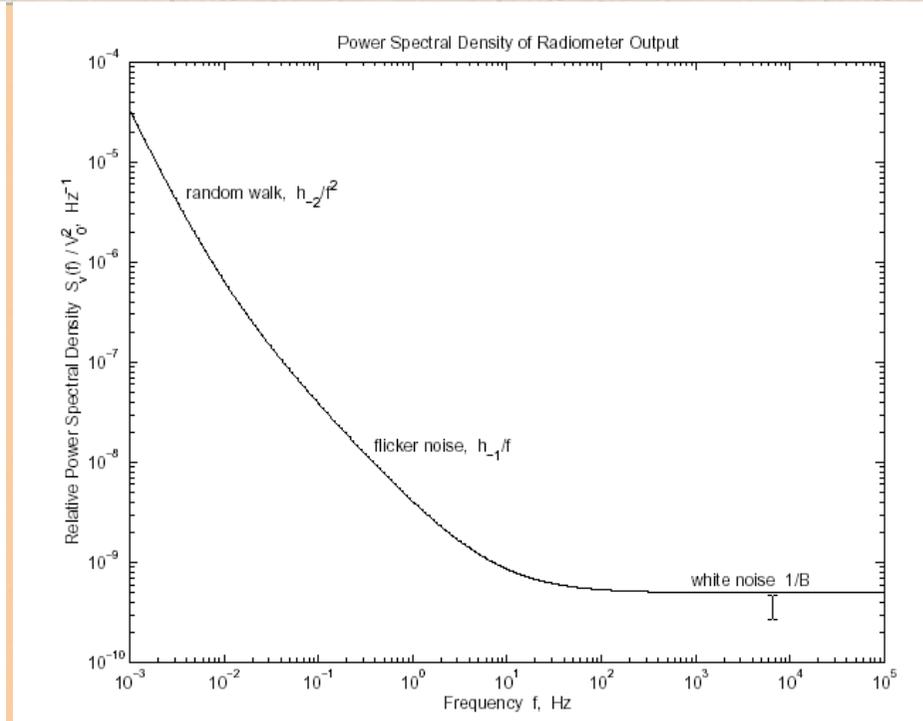
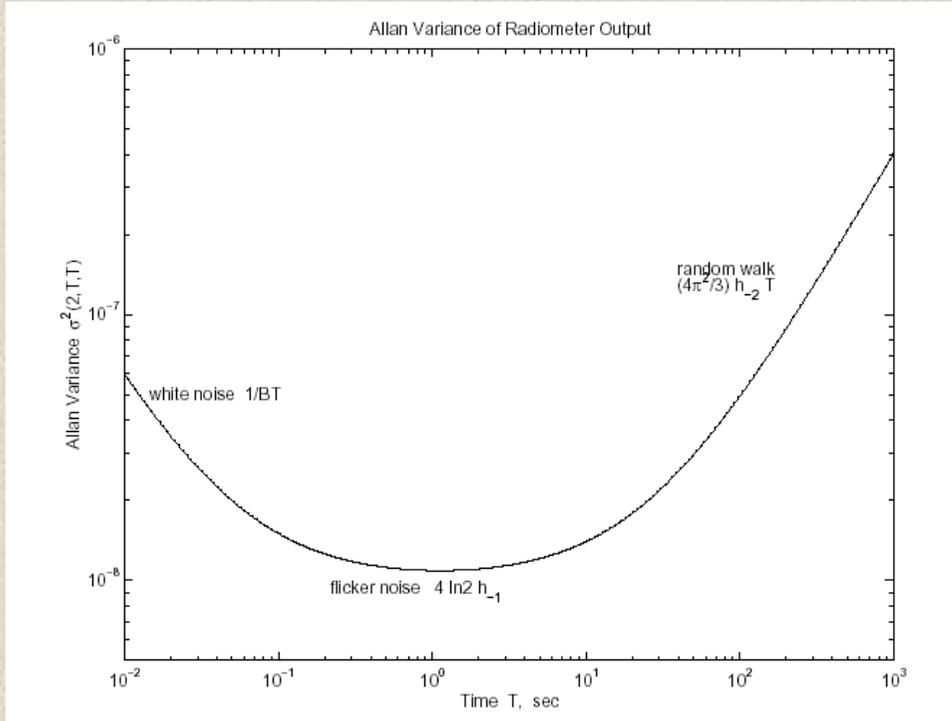
## ■ SNGF (units: $1/\sqrt{\text{Hz}}$ , sometimes $1/\text{Hz}$ ):

- Spectrum of Normalized Gain Fluctuation
- Frequency (video) domain
- Engineer's choice

$$S(0) = \text{mean}(G(t)) = 1$$

$$\int_{f_{\min}}^{f_{\max}} |S(f)|^2 \cdot df = \text{var}(G(t))$$

# Allan Variance vs. SNGF



$$\sigma^2(\tau) = \int_{-\infty}^{+\infty} S(\nu) \left| 4 \frac{\sin^4(\pi\tau\nu)}{(\pi\tau\nu)^2} \right|^2 d\nu$$



# Conversion of SNGF to Allan Variance

- SNGF in  $1/\sqrt{\text{Hz}}$  units:

$$S(f) = b \cdot f^{-\alpha} \Leftrightarrow \sigma^2(\tau) = K_{\alpha} \cdot b^2 \cdot \tau^{2\alpha-1}$$

(valid for  $-\frac{1}{2} < \alpha < \frac{3}{2}$ )

- Ambiguity:

Note that the inverse mapping is not unique (more information in SNGF)

However in practical cases with random signals the inverse conversion is “valid”

$$K_1 = \frac{2}{3} \cdot \pi^2$$

$$K_{3/4} = \frac{32}{15} \cdot \pi \cdot (\sqrt{2} - 1)$$

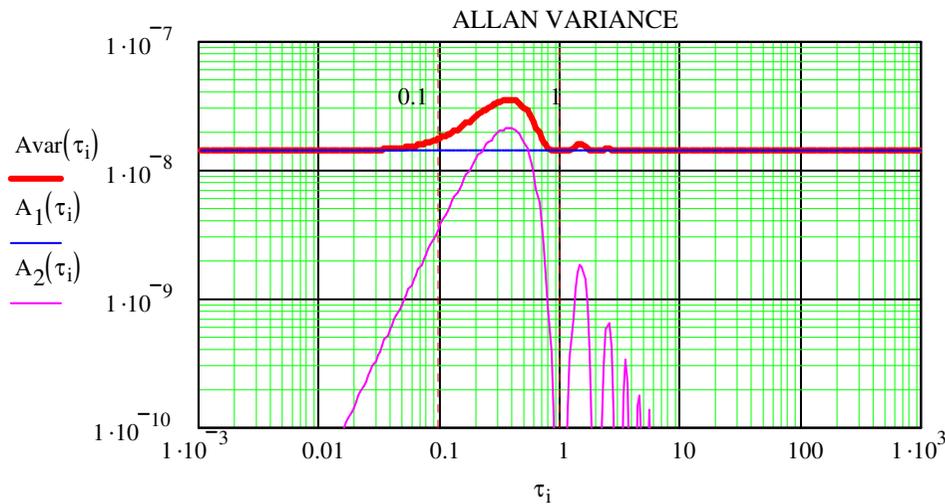
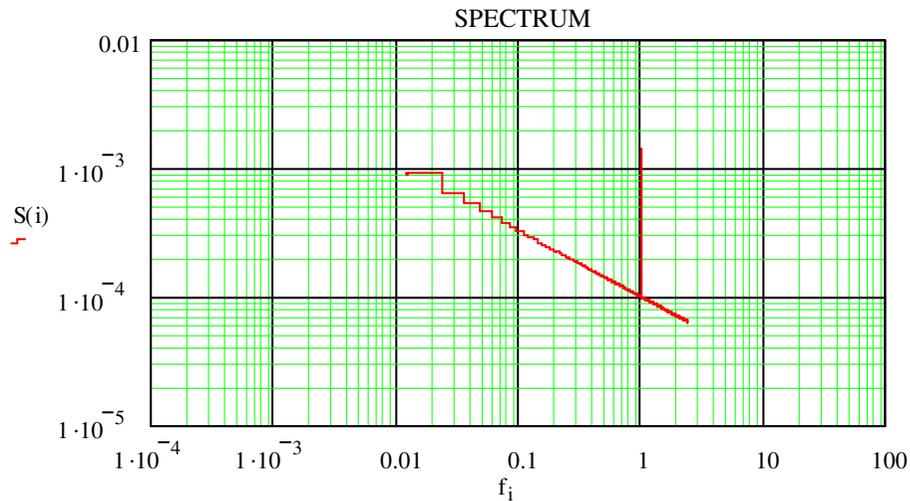
$$K_{1/2} = 2 \cdot \ln(2)$$

$$K_{1/4} = \frac{4}{3} \cdot (2 - \sqrt{2})$$

$$K_0 = \frac{1}{2}$$

$$K_{-1/4} = \frac{1}{2\pi} \cdot (4 - \sqrt{2})$$

# Why using SNGF ?



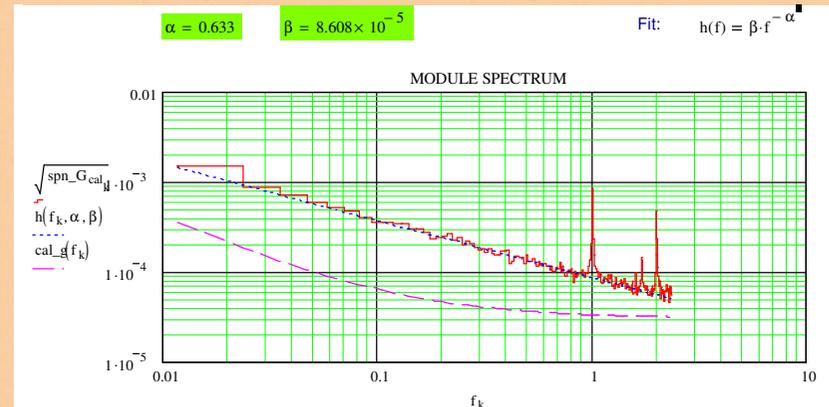
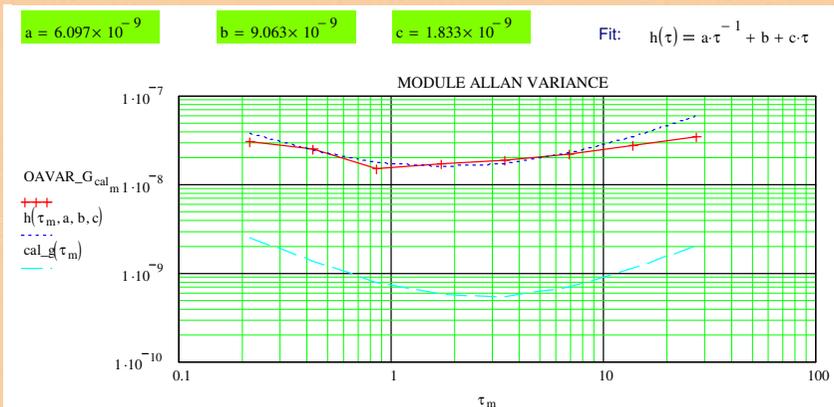
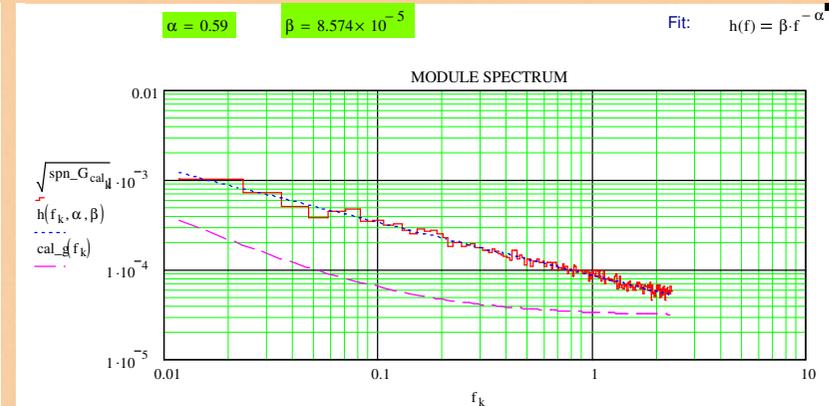
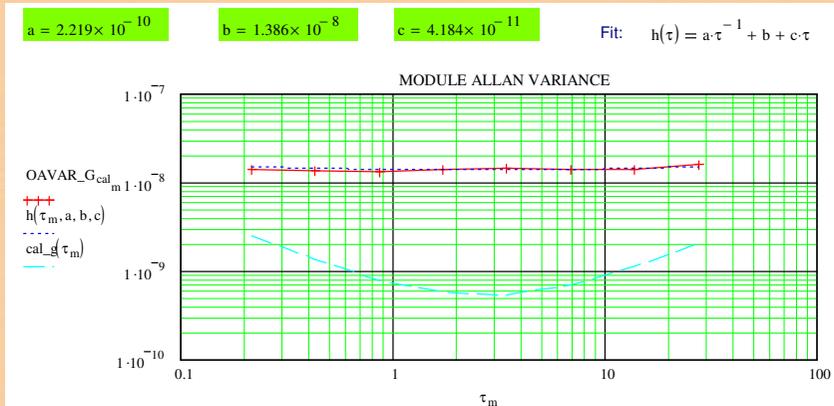
## ■ Example:

- 1 Hz gain oscillation (typical refrigerator cycle)
- $A=2E-4$  (0.0017 dB pp)
- Typical InP amplifier fluctuation

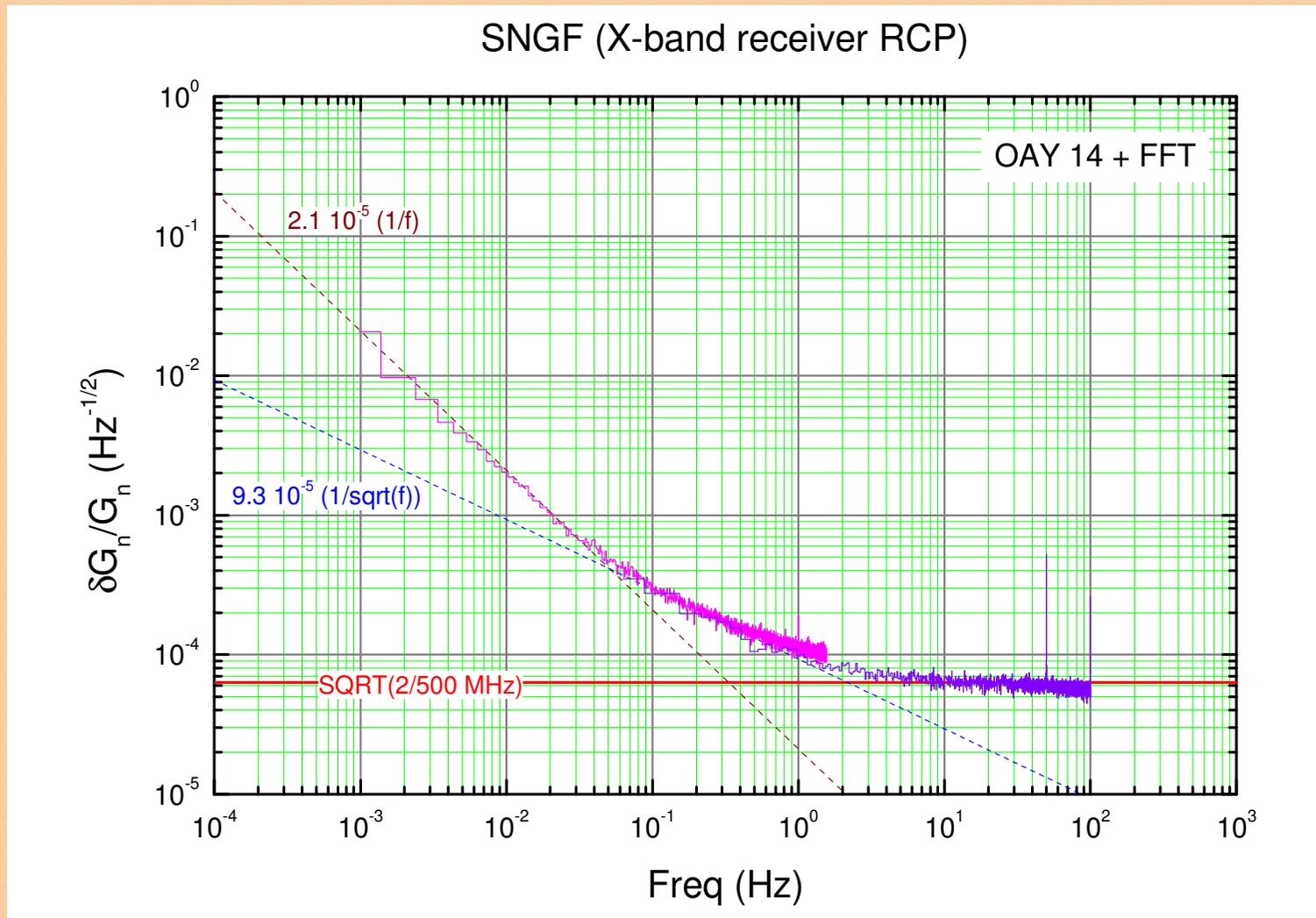
## ■ Result:

- Allan Variance “bump” spread over time (0.1-1 sec)

# Why using SNGF ? (real world)

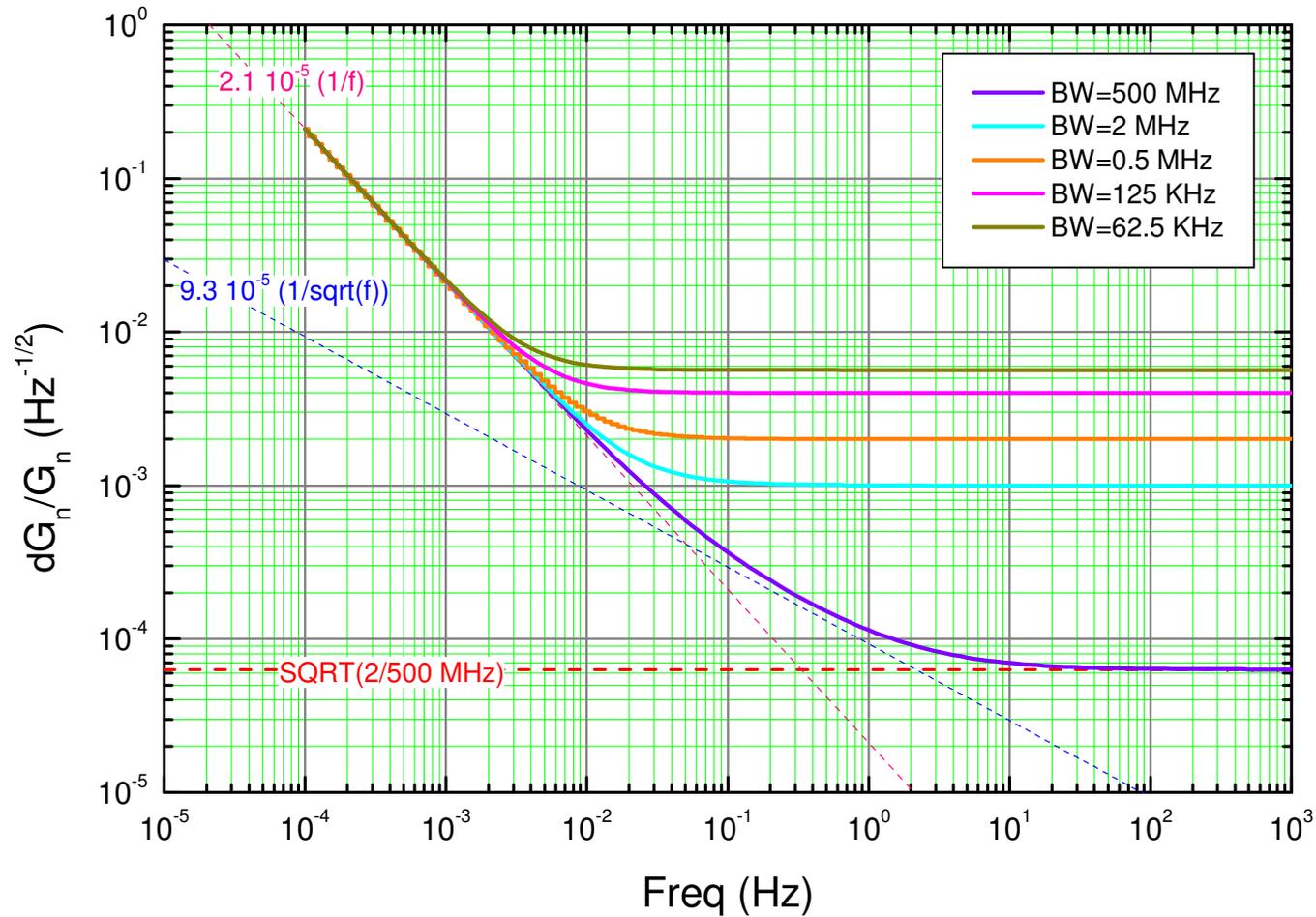


# Real receiver: CAY 40m

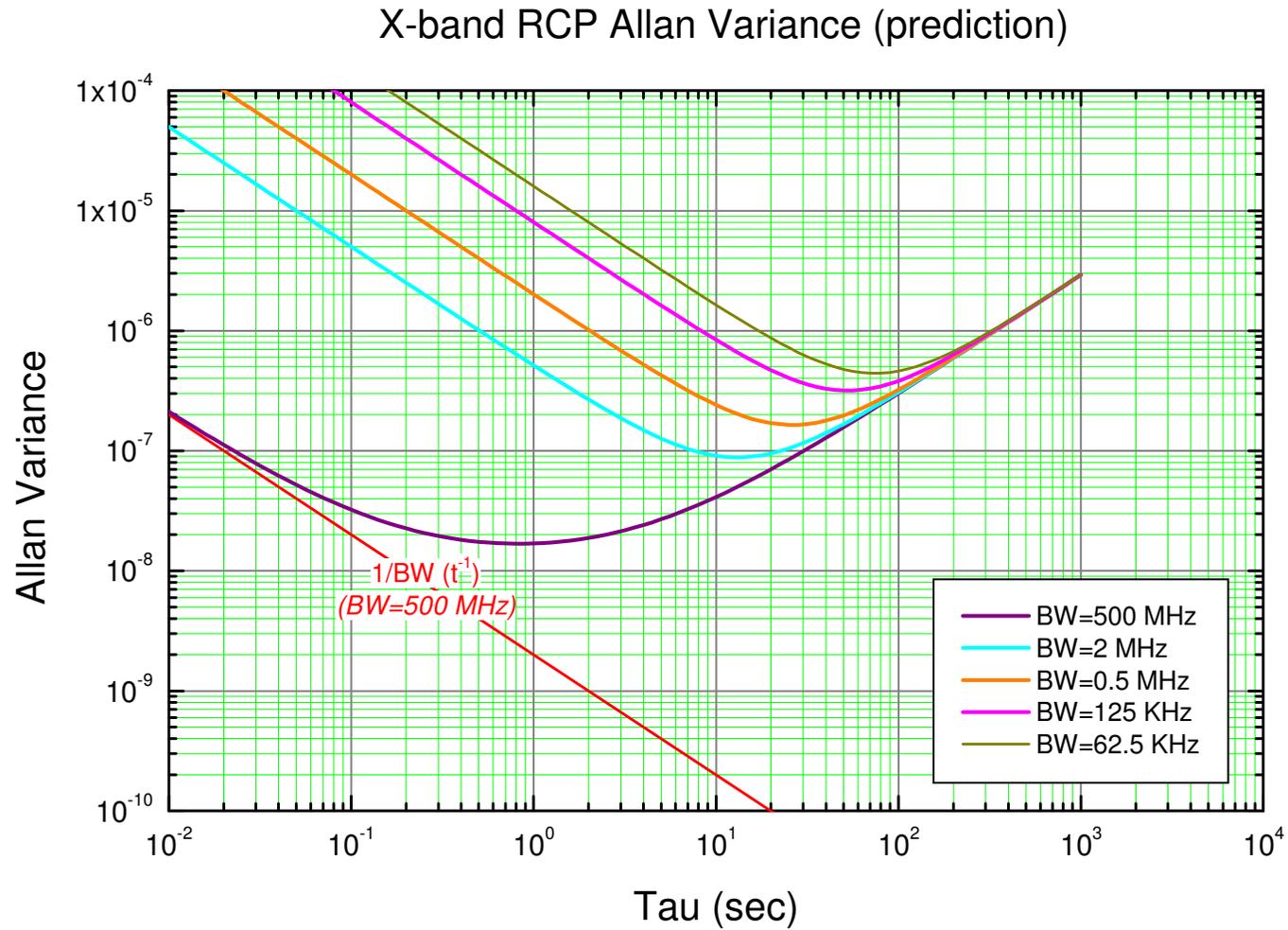


# Real receiver: CAY 40m VLBI

SNGF (X-Band receiver, RCP, prediction)



# Real receiver: CAY 40m VLBI



# Specs of Gain Fluctuations (examples)

## ■ HERSCHEL:

- Far Infrared and Submillimeter 3.5 m Telescope orbiting in L2 with 3 cryogenic instruments
- HIFI: Heterodyne Instrument for the Far Infrared with 7 dual polarization submillimeter SIS and HEB receivers
- Low noise, wide band 4-8 GHz cryogenic IF preamplifiers
- Specification for cryo amplifiers:

$$\Delta G_n = 1.4 \cdot 10^{-4} \frac{1}{\sqrt{\text{Hz}}} @ 1 \cdot \text{Hz}$$

## ■ ALMA:

- Atacama Large Millimeter Array (USA, Europe, Japan), 64 Antennas, 35-850 GHz
- 4-8 and 4-12 GHz cryogenic IF preamplifiers (bands 7 and 9)

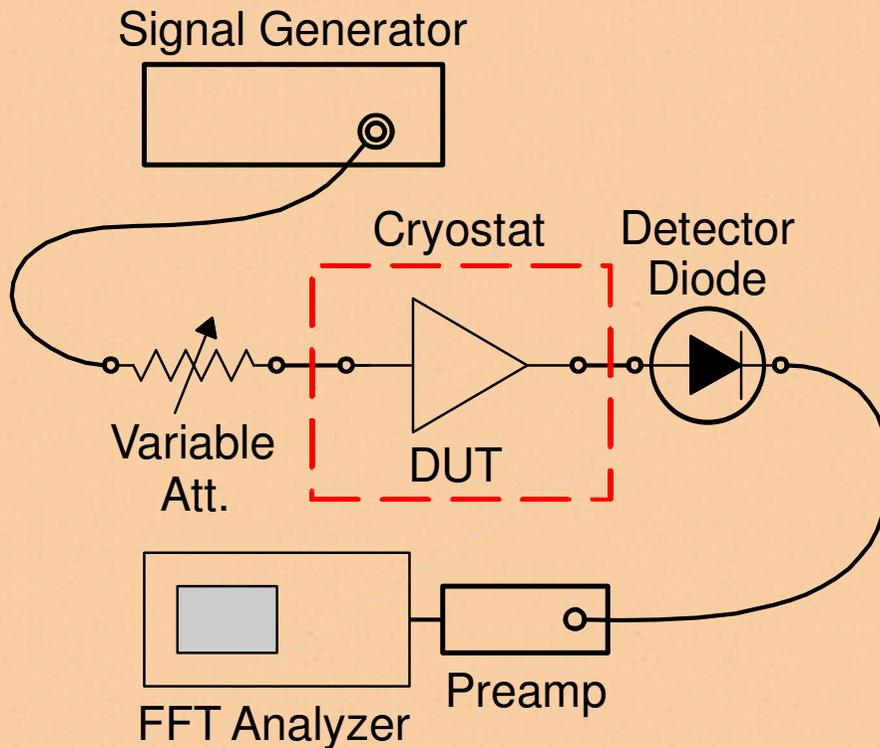
$$\begin{cases} \sigma^2(\tau) \leq 2 \cdot 10^{-8} & 0.05 \leq \tau \leq 100 \text{ (sec)} \\ \sigma^2(\tau) \leq 2 \cdot 10^{-7} & 100 < \tau \leq 300 \text{ (sec)} \end{cases}$$

$$\Delta G_n = 8.4 \cdot 10^{-5} \frac{1}{\sqrt{\text{Hz}}} @ 1 \cdot \text{Hz}$$

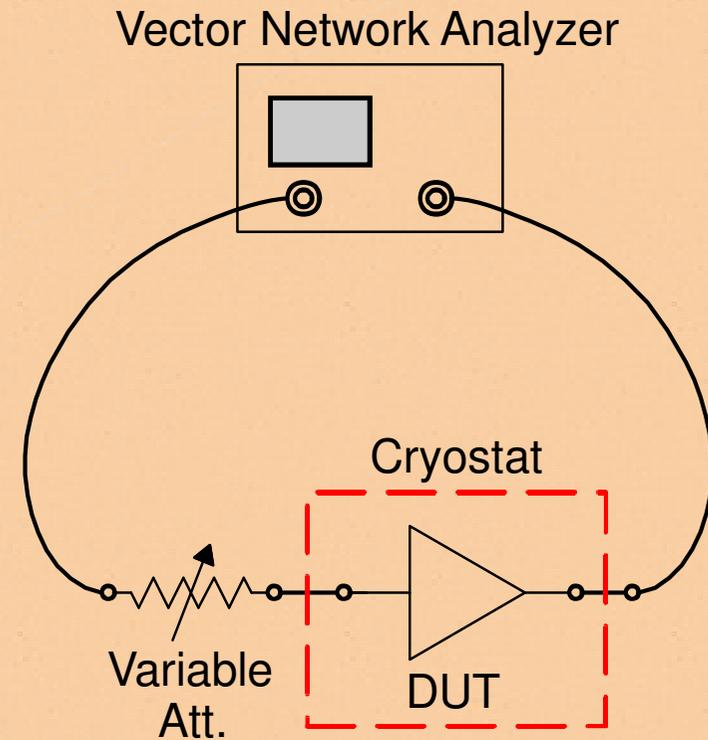
# Gain Fluctuation Measurements

## Measurement of individual amplifiers

### System A

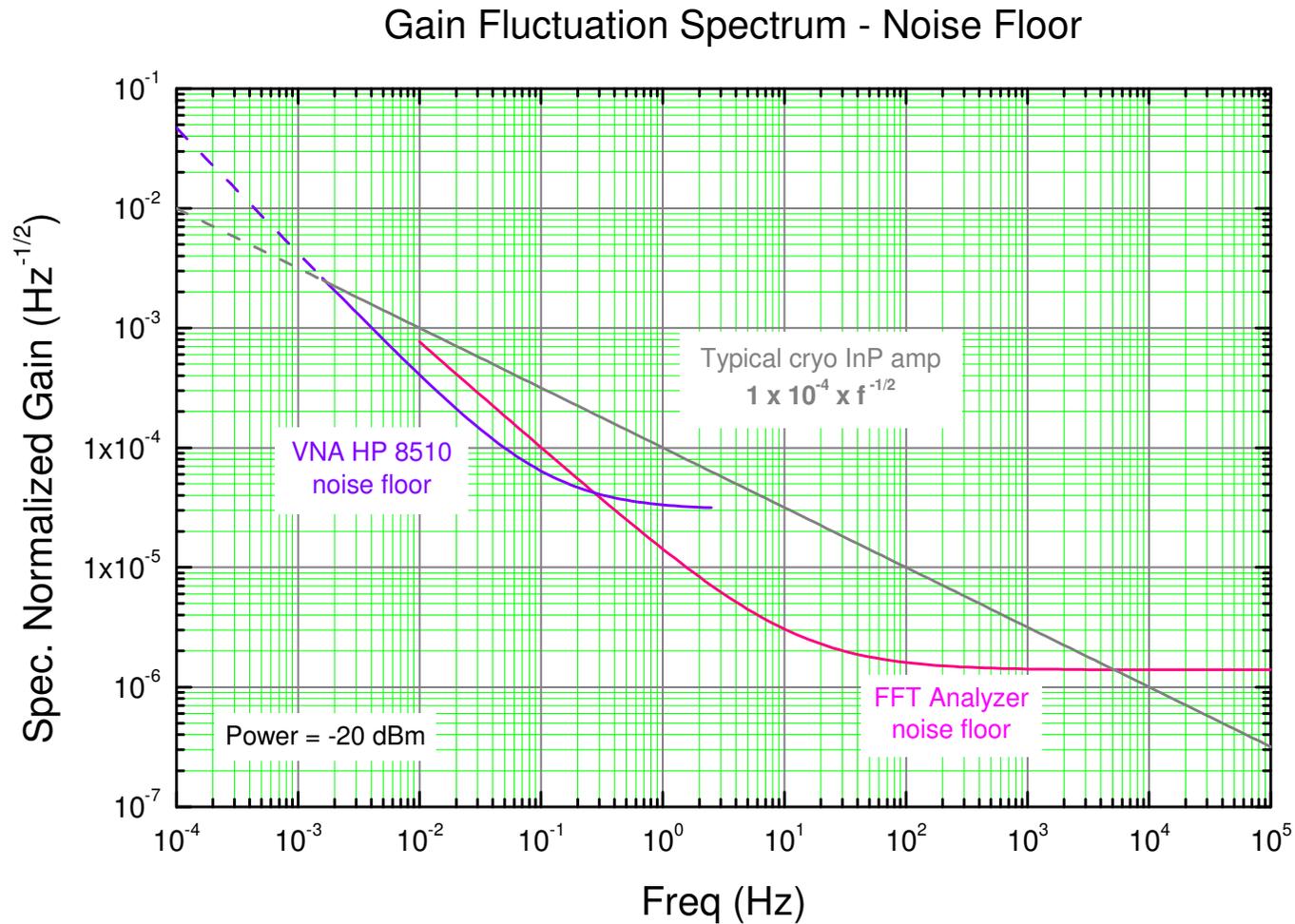


### System B



Details in [ALMA memo 560](#)

# Measurement Systems

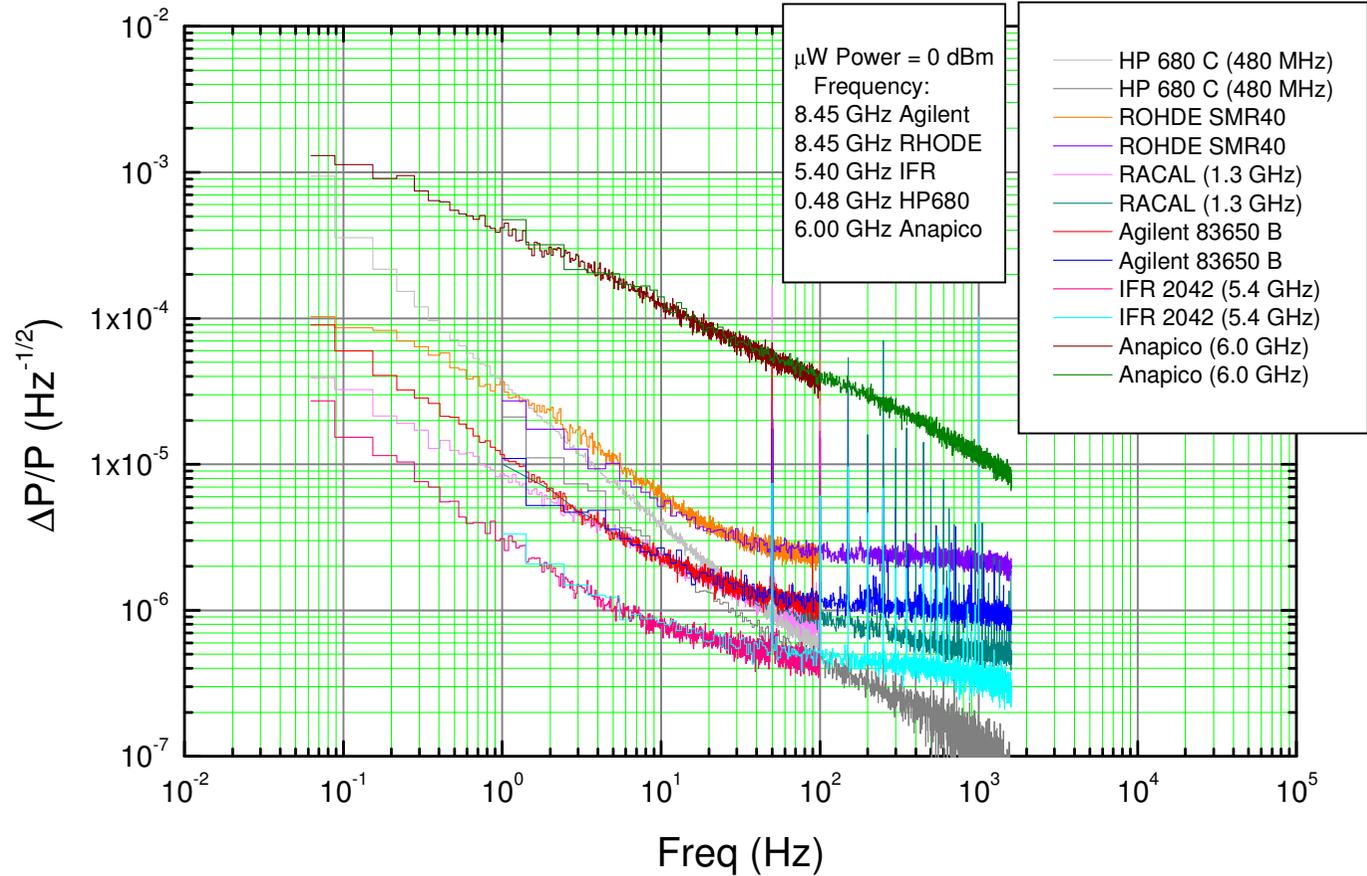




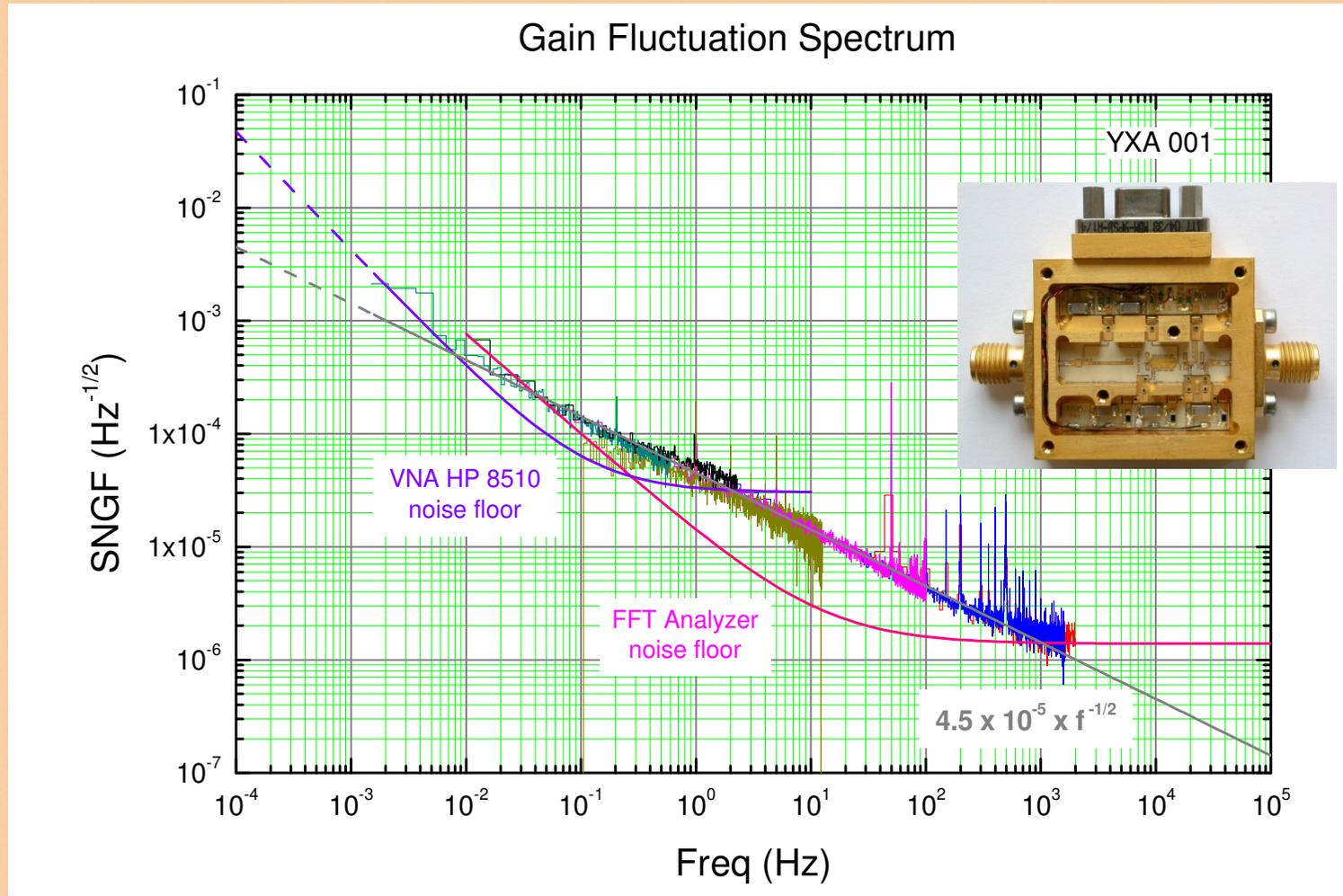
# Performance of Signal Generators



## Normalized Power Fluctuation Spectrum



# Measurement example



# Lessons Learned

- Classical approach: noise input
  - High MW gain needed (many amp. stages or complete radiometer)
  - Sensitivity limited at high audio frequencies (noise bandwidth)
- This approach: single tone
  - Good generator or VNA needed
  - Characterization of individual amplifiers (even a single stage!)
  - Information at different input frequencies in the band
  - Good sensitivity at high audio frequency (generator + FFT)
  - Highly immune to interferences (VNA)
  - Information on phase noise possible (VNA)
    - New VNAs (PNA) not as good as old models (8510) for stability measurements
- Measurements are slow and should be done very carefully
  - Stable temperature in the lab
  - No cell phones!

# Gain Fluctuation Origin

## ■ Origins

- External

- Power supply

- Temperature variations (refrigerator cycle)

- Microphonics (vacuum pumps)

- Internal

- Intrinsic to the devices

## ■ Typical Intrinsic Spectrum of Cryogenic Amplifiers

$$S(f) = b \cdot \left( \frac{1 \cdot \text{Hz}}{f} \right)^\alpha \quad \alpha \approx 1/2$$

# Is the Power Supply important?

## ■ Possibilities:

- Constant  $V_d$ ,  $V_g$
- Constant  $I_d$ ,  $V_g$  (variable  $V_d$ )
- Feedback: Constant  $V_d$ ,  $I_d$  (controlled  $V_g$ )

## ■ Power supply must be low noise

- Examples: Agilent N3280 or classical NRAO feedback supply
- Typically the gain fluctuation caused by noise from the power supply is more than two orders of magnitude below the observed values in cryogenic amplifiers

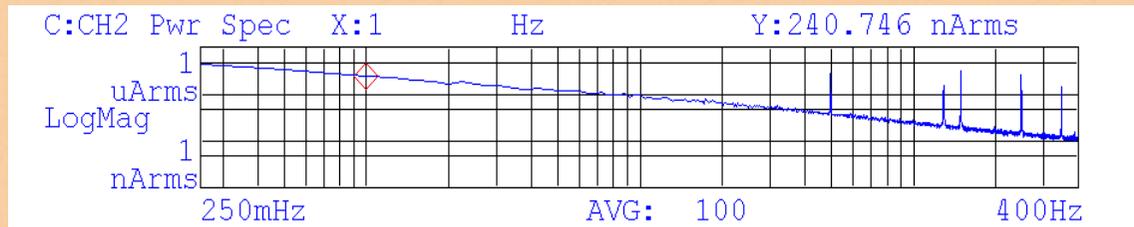
## ■ Best results always obtained with feedback supply

- but not much improvement at cryogenic temperature

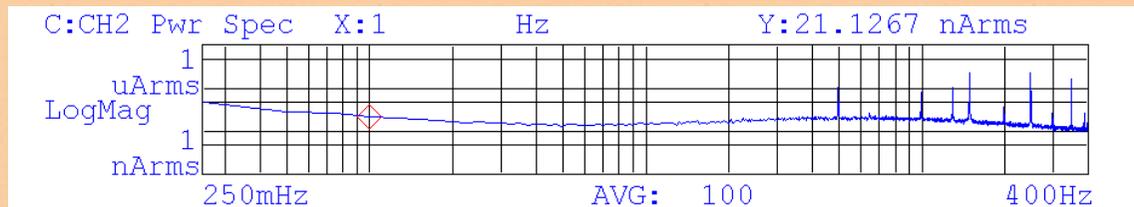
## ■ Worst results obtained with constant $I_d$ , $V_g$



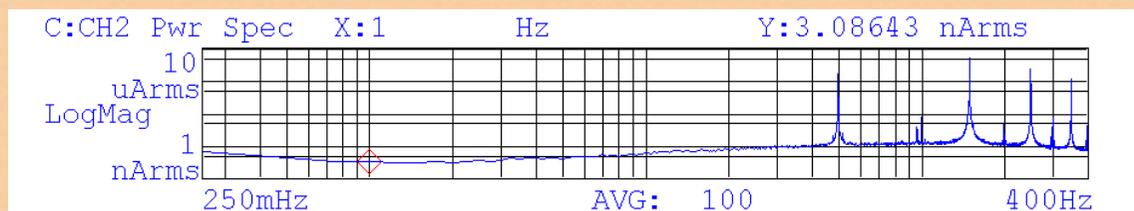
# Illustration of the reduction of Id fluctuation with Feedback Power Supply at T=15 K



YCF 4001 Vd=1.10  
Id=3 Rd=172 T=14 K,  
Constant Vd, Vg



YCF 4001 Vd=1.10  
Id=3 Rd=172 T=14 K,  
Constant Id, Vg



YCF 4001 Vd=1.10  
Id=3 Rd=172 T=14 K,  
Feedback Supply

# Coherence of Gain-Id fluctuations (ambient)

## Measurements on 1 stage amplifier (YCF4001)

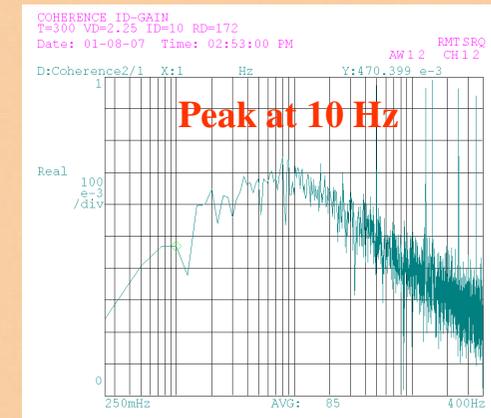
### Vd constant

Vd (supply)	Vd(drain)	$\Delta G_n$ (1Hz BW=1Hz)	$\Delta I_d$ (nA) (1Hz BW=1Hz)	Coherence
2.00	0.28	8.7 E-5	204	0.95
2.25	0.53	1.5 E-5	260	0.47
2.50	0.78	<1 E-5	160	0.09

### Id constant and feedback supply

Vd (supply)	Vd(drain)	$\Delta G_n$ (1Hz BW=1Hz)	$\Delta I_d$ (nA) (1Hz BW=1Hz)	Coherence
2.25	0.53	4.4 E-5	41	Id=constant
2.25	0.53	<1 E-5	9.0	Feedback sply

- **Very strong coherence at low drain voltage**  
(linear part of Id-Vd curve)
- **Only partial coherence at the “optimum” bias**
- The DC current fluctuation does not follow the same pattern
- **Worst results** obtained with **constant Id** and **Vg**
- **Best results** obtained with **feedback supply**



# Coherence of Gain-Id fluctuations (cryogenic)

## Measurements on 1 stage amplifier (YCF4001)

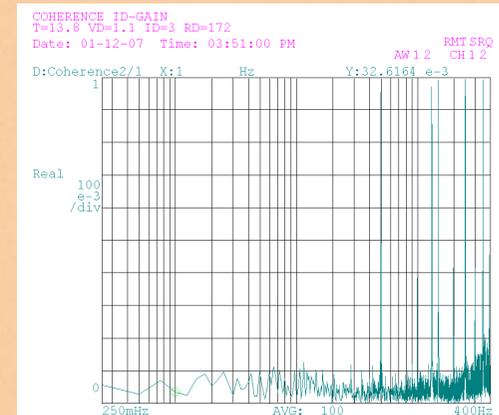
### Vd constant

Vd (supply)	Vd(drain)	$\Delta G_n$ (1Hz BW=1Hz)	$\Delta I_d$ (nA) (1Hz BW=1Hz)	Coherence
1.30	0.784	16 E-5	436	~-0.20
1.10	0.584	14 E-5	480	~-0.05
0.90	0.384	6 E-5	116	~-0.05

### Id constant and feedback supply

Vd (supply)	Vd(drain)	$\Delta G_n$ (1Hz BW=1Hz)	$\Delta I_d$ (nA) (1Hz BW=1Hz)	Coherence
1.10	0.584	12 E-5	6.0	Feedback sply
0.90	0.384	4.5 E-5	4.8	Feedback sply
1.10	0.584	16 E-5	42	Id=constant

- **Low coherence** (only observed at high Vd)
- **Higher values** of gain fluctuation and Id fluctuation at cryogenic temperature
- **No (significant) improvement** with feedback supply



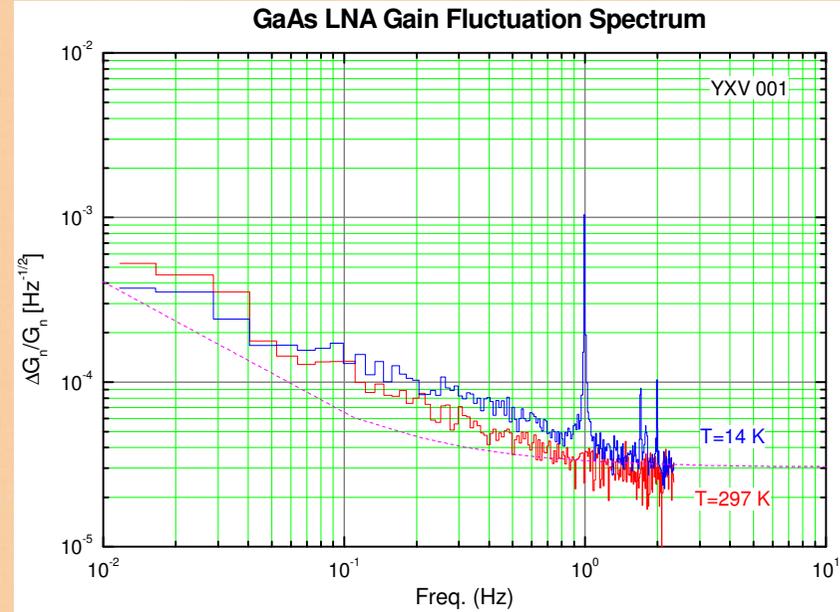
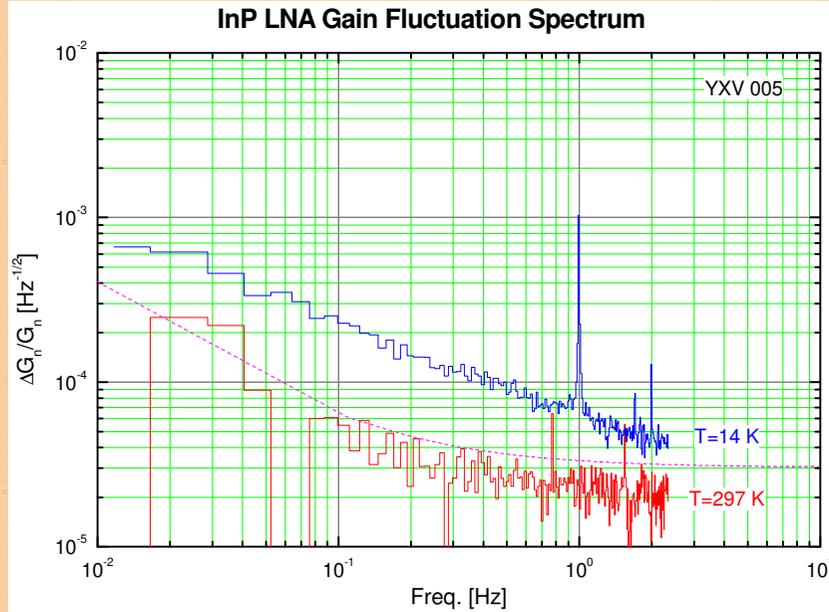
## What can be done?

- Unfortunately DC and MW fluctuation appear to be non-coherent at optimum bias and cryo temperature → power supply does not help much
- Similar to problem with  $R_{ds}$  and  $G_m$  (different values at low and MW, related with trap time constants)
- Use of “pilot” signals has been suggested sometimes (E. Wollak 1995, S. Weinreb)
- In this work: modulation at audio frequency of bias supply (low level, ~60 KHz, cryogenic temperature) for sampling  $G_m$  was tried. A coherence of 0.3 with was found
- More work needed

# Intrinsic Gain Fluctuations Dependence

- Improvements in Noise
  - Shorter gate length
  - HEMTs
  - InP substrate
  - Cryogenic Temperature
- Improvements in Gain Stability
  - Larger gate area
  - Non HEMT
  - GaAs substrate
  - Ambient Temperature
- Conclusion:
  - Advances in the reduction of Noise Temperature have contributed to increase Gain Fluctuations

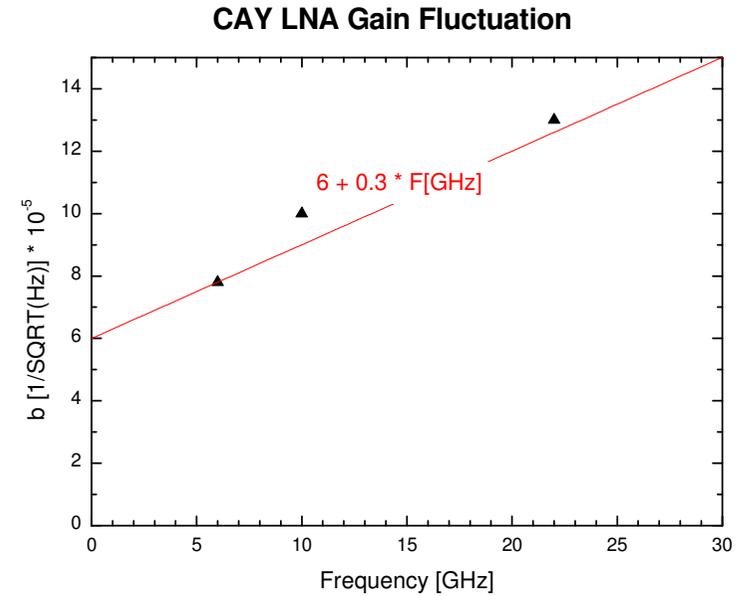
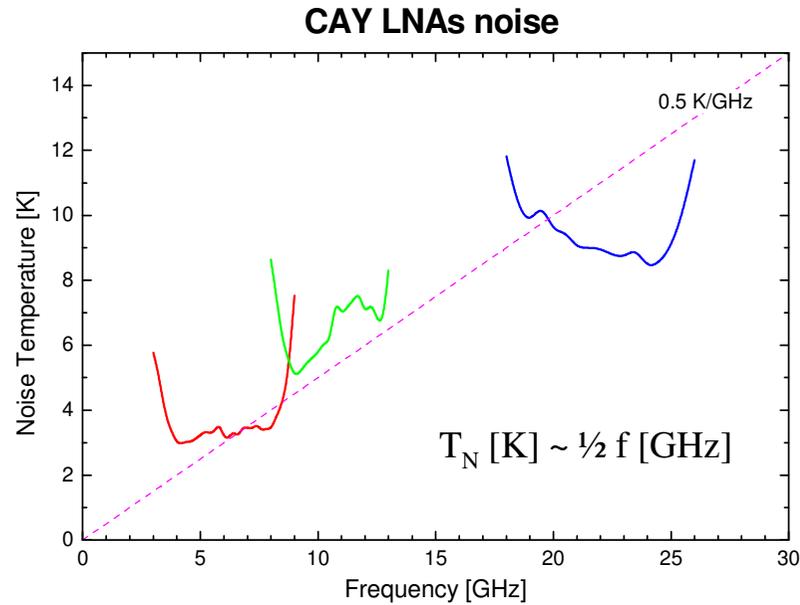
# Material (InP vs. GaAs) and Temperature (amb. vs. cryo.)



DEVICES		NOISE TEMPERATURE [K]			GAIN FLUCTUATIONS [ Hz <sup>-1/2</sup> ]		
TECHNOLOGY	MODEL	297 K	14 K	VARIATION	297 K	14 K	VARIATION
GaAs	FHX13X 200×0.25μm	99	13.6	÷ 7.3	2.0E-5	3.4E-5	× 1.7
InP	NGST 160 160×0.1μm	96	6.5	÷ 14.8	(1.3E-5)	9.4E-5	× (7.2)
	ETH 200 200×0.2μm	108	6.7	÷ 16.1	(1.0E-5)	7.0E-5	× (7.0)
GaAs – InP Variation [times]		None	÷ 2.1	–	÷ (1.7)	× 2.8	–



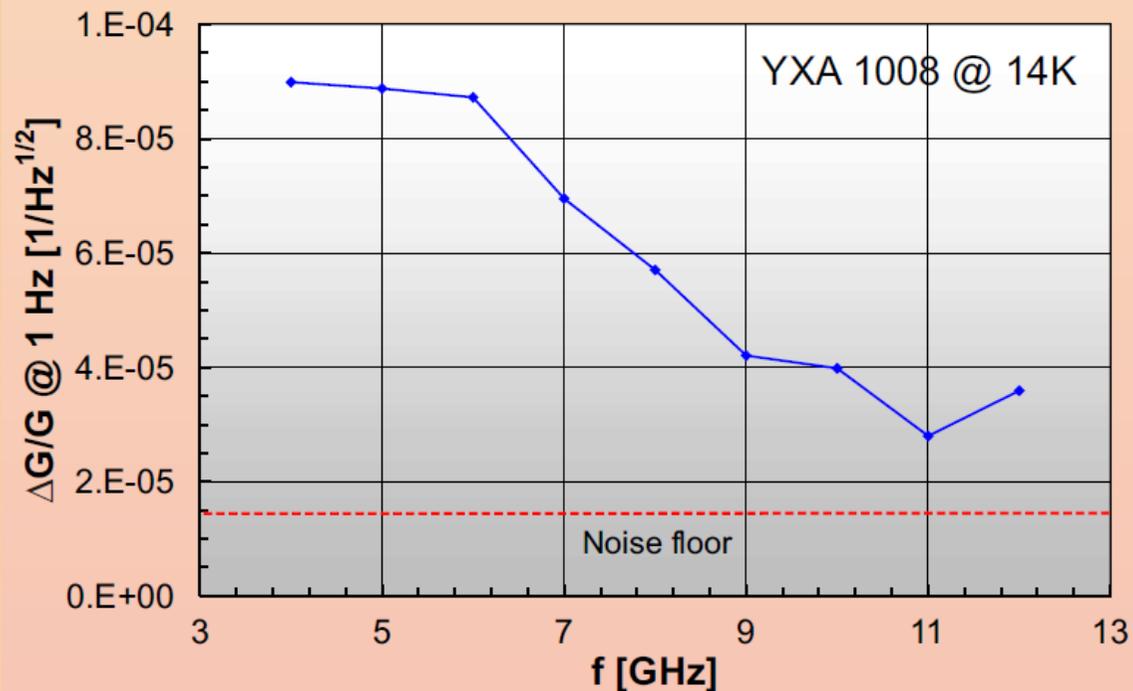
# Frequency Band



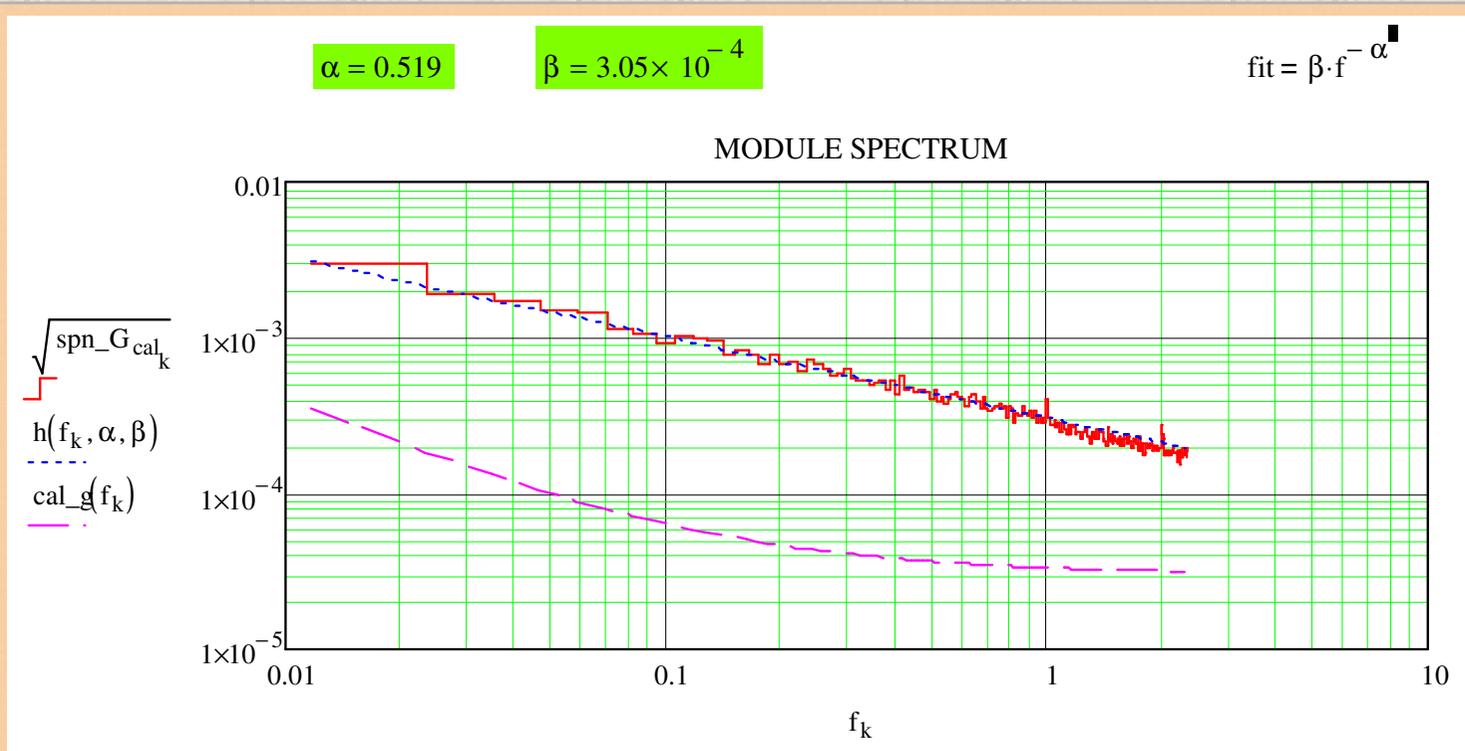
AMPLIFIER	BAND [GHz]	STAGES	DEVICES	NOISE [K]	GAIN [dB]	GAIN FLUCTUATIONS $b [Hz^{-1/2}]$	$\alpha$
YCA 2	4 - 8	3	2 × NGST CRYO4	3.5	39.2	7.8E-5	0.64
YXF 0	8-12	2	NGST 160 + FHX13X	6.5	21.9	10E-5	0.58
YK22	18 - 26	3	NGST CRYO3 + 2 × NGST 160	8.9	26.1	13E-5	0.43

# Different Frequencies in the Band

**Frequency dependence:** Fluctuations @ 1 Hz measured with CW vary significantly with the input frequency. This may be due to the dominant role of the transistors transconductance  $g_m$  in the gain fluctuations. Monte Carlo simulations performed show that variations of  $g_m$  produce variations of gain with similar frequency dependence.  $C_{gs}$  will produce gain variations in the opposite sense.

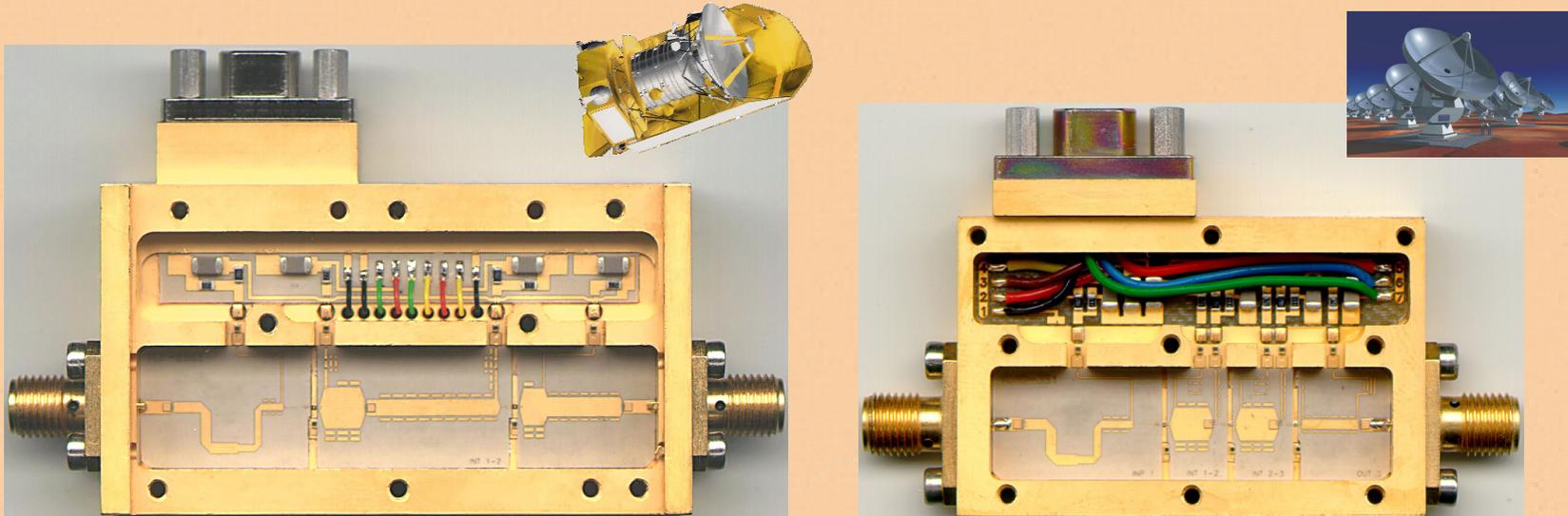


# Defective Materials



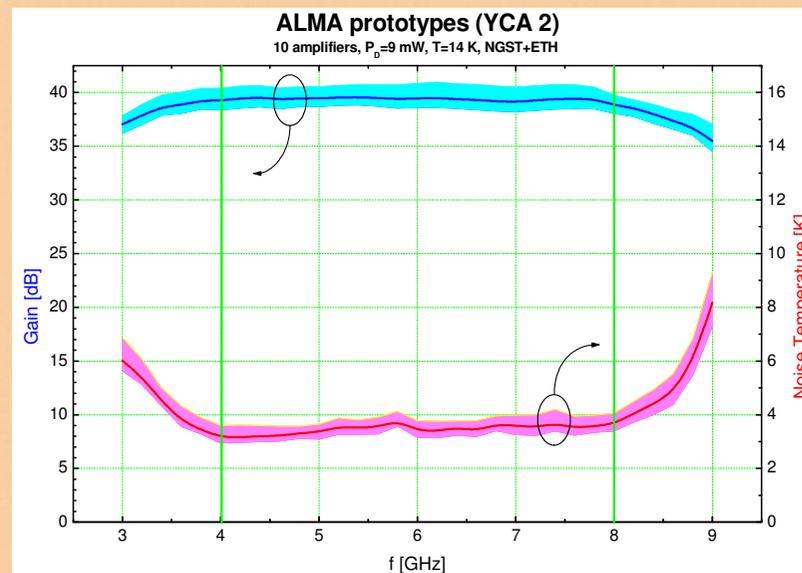
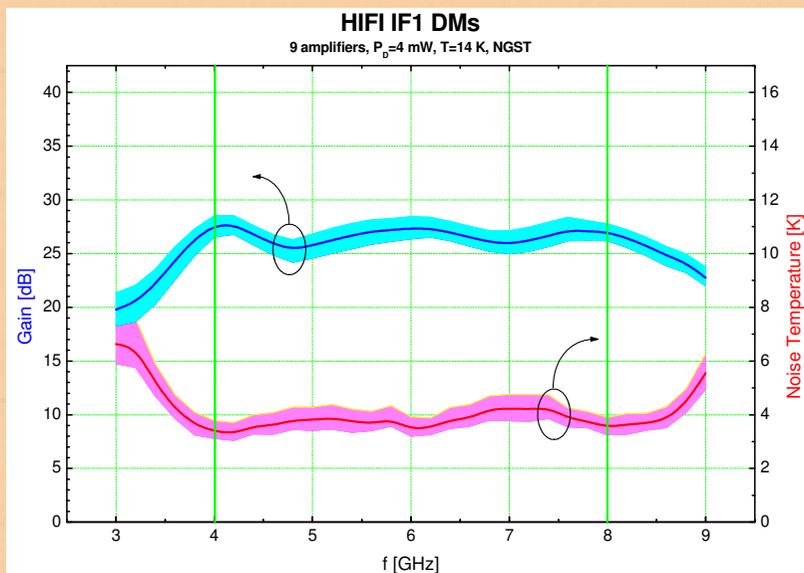
- Manchester  $0.13 \times 200 \mu\text{m}$  Run March 2011
- Bad material, ~ 4 times more fluctuation
- Noise was almost normal

# Statistical Analysis: Results of long series (1)



- Analyzed two series of 9 and 10 state-of-the-art 4-8 GHz cryogenic amplifiers designed and built at CAY with microstrip hybrid technology:
  - Development models for Herschel space telescope (all cryogenic LNAs for HIFI)
    - 2 stages of InP NGST transistors
    - Design constrained by space qualification
  - Pre-production phase of ALMA (cryogenic LNAs for the European contribution)
    - 3 stages of InP NGST and ETH transistors.
    - Based on HIFI design, with more degrees of freedom

# Statistical Analysis: Results of long series (2)



AMPLIFIER YCF	GAIN FLUCT. @ 1 Hz ( $b$ )	SPECTRAL INDEX ( $\alpha$ )
6004	8.62E-05	0.754
6005	8.88E-05	0.706
6006	8.58E-05	0.332
6007	14.5E-05	0.430
6009	10.0E-05	0.726
6010	8.69E-05	0.415
6011	11.4E-05	0.324
6012	9.03E-05	0.429
6014	10.2E-05	0.762

AMPLIFIER YCA	GAIN FLUCT. @ 1 Hz ( $b$ )	SPECTRAL INDEX ( $\alpha$ )
2003	7.43E-05	0.668
2004	7.65E-05	0.690
2005	7.91E-05	0.606
2006	7.51E-05	0.731
2007	7.04E-05	0.640
2008	8.72E-05	0.647
2009	8.61E-05	0.633
2010	8.39E-05	0.588
2011	7.21E-05	0.639
2012	7.13E-05	0.606



## Statistical Analysis: Results of long series (3)

HIFI AMPLIFIERS

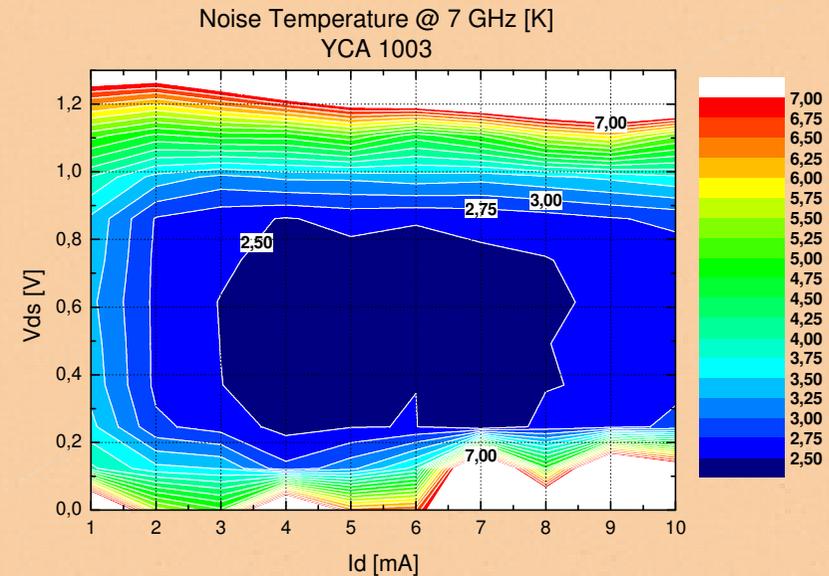
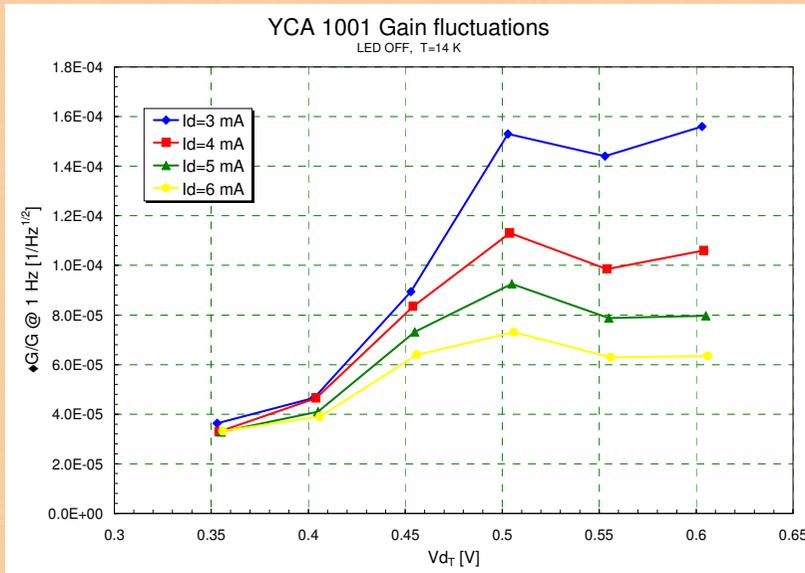
$x$	MIN.	MAX.	$\bar{x}$	$\sigma/\bar{x}$
$T_N$ [K]	3.4	4.2	<b>3.78</b>	<b>0.076</b>
$b$ [Hz <sup>-1/2</sup> ]	8.58E-5	14.5E-5	<b>9.99E-5</b>	<b>0.194</b>
$\alpha$	0.321	0.762	0.542	0.351

ALMA AMPLIFIERS

$x$	MIN.	MAX.	$\bar{x}$	$\sigma/\bar{x}$
$T_N$ [K]	3.2	3.8	<b>3.46</b>	<b>0.051</b>
$b$ [Hz <sup>-1/2</sup> ]	7.04E-5	8.72E-5	<b>7.76E-5</b>	<b>0.080</b>
$\alpha$	0.588	0.731	0.645	0.066

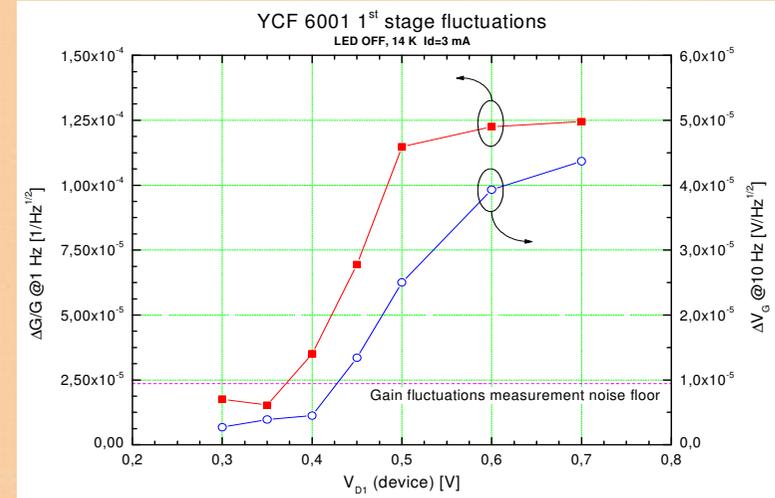
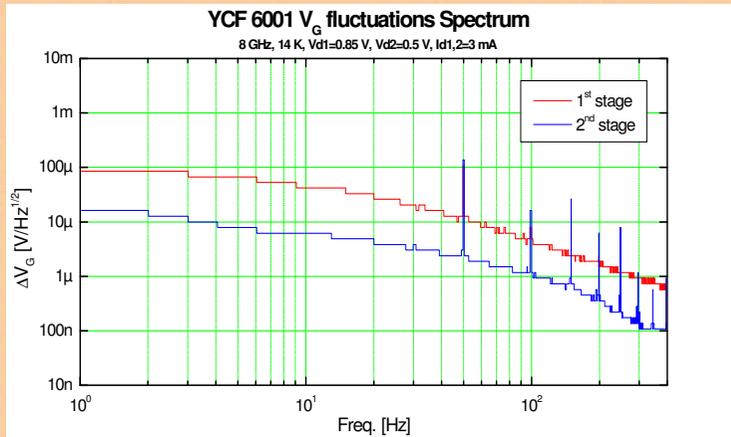
- Mean values of noise and fluctuations are similar in both series
- Excellent repeatability in noise and gain
- Greater dispersion of gain fluctuations results (value @ 1 Hz and spectral index)
- HIFI dispersion in fluctuations significantly wider than ALMA
  - Bias conditions homogeneous for both series
  - Different transistor batch used for each series of amplifiers
- **Some transistor batches exhibit high scattering of gain fluctuations within devices of the same batch not shown in noise results**
- **Observed also a significant batch to batch variation**

# Bias Point (1)

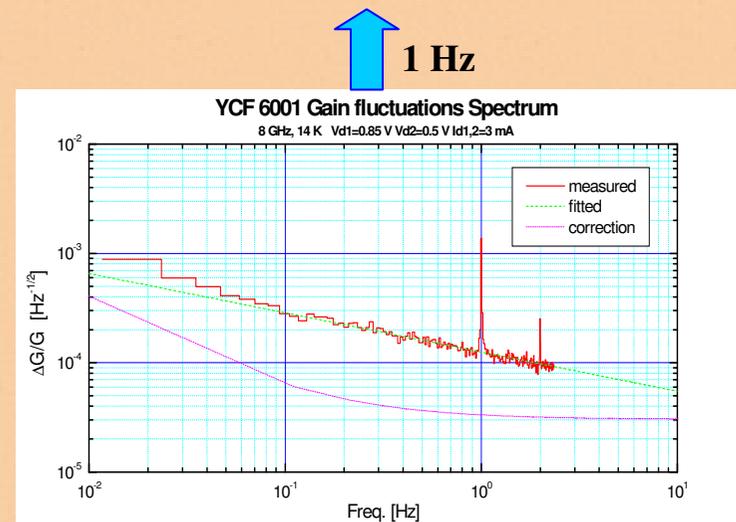


- Tested the variation of gain fluctuations and noise with  $V_d$ ,  $I_d$ 
  - Found a steep change in gain fluctuations around 0.4-0.5 V
  - Noise (and gain) are much more insensitive to bias changes
  - High fluctuation zones could be avoided with no penalty in noise or gain
- Gain fluctuations  $\uparrow$  when  $I_d \downarrow$

# Bias Point (2)



- Tested also fluctuations of gate voltage with HP35670A
  - Found similar bias dependence
- Simple measurements of voltage fluctuations may help detecting sensitive bias regions
- Correlation with gain fluctuations for different devices useful for pre-selecting least fluctuating devices from a batch



## Conclusions

- The reduction in noise and the increase in bandwidth of modern cryogenic amplifiers have made more prominent the problem of gain fluctuations
- Data on gain fluctuations of cryogenic amplifiers collected over the years
- Systematic measurement with a VNA are possible, although very time consuming
- The reduction (or at least some control) of the gain fluctuations is possible with an adequate selection of devices and bias
- Lack of theoretical model

END

