Radio polarisation calibration

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Abstract

High quality calibration and high dynamic range imaging are required by many key science projects of future radio interferometric telescopes. This is especially the case for the study of (polarised synchrotron emission sources and) cosmic magnetism which requires full Stokes information. High dynamic range imaging requires deconvolution as well as full Stokes calibration of instrumental and atmospheric effects which are generally time variable and direction dependent. This mainly includes achieving high polarisation calibration accuracies by the ability of handling and removal of systematics in order to be able to make use of the high sensitivities and the large field-of-views of the new generation of instruments. In this Memo we give an introduction and overview of the state of the art of polarisation calibration.

1 Introduction

Up-coming and future instruments, such as phased arrays, will due to their inherent strong instrumental polarisation, wide bandwidths and high channel numbers require full polarisation calibration over wide fields of view in order to achieve good imaging. Assumptions and approximations made by scalar selfcal methods - still widely applied to date - do no longer hold for such instruments.

The instrumental sensitivity of an aperture synthesis array is commonly stated by:

$$\Delta S \propto T_{sys}/\sqrt{(}\Delta\nu\Delta t n(n-1))/A, \quad (1)$$

with T_{sys} being the temperature of the system, $\Delta\nu$ being the total bandwidth and Δt the total integration time of an observation. n is the number of elements with the collecting area A in the interferometric array. The signal-to-noise ratio with which a point source in the field-of-view can be observed is then proportional to the inverse of equation 1. However, this neglects systematic and instrumental effects by which the data is unavoidably affected. There are time variable and direction dependent effects which have to be accounted for. Instrumental polarisation leakages can affect co-polar visibilities and thus reduce significantly the achievable dynamic range. Thus, in order to achieve high dynamic range imaging and make full use of up-coming facilities an accurate full Stokes polarisation calibration over the entire field of view has to be performed. Moreover, radio polarisation measurements provide additional information to insights gained from total information about emission and propagation. Thus, (polarisation) calibration is of utmost importance to achieve high dynamic range images of the Stokes parameters. A convenient and sufficient way is to use the measurement equation formulation in order to describe the observed polarisation signal.

Instruments like ASKAP, MeerKAT, LOFAR and finally the SKA will have unprecedented sensitivities which are due to their large number of elements and thus large collecting areas up to several orders of magnitude higher than current instruments can provide. The increased sensitivity, the wide bandwidths with large number of sub-channels and the large spatial extent of the new instruments require us to move forward into new calibration regimes. Further, extremely high dynamic range imaging requires that we can no longer assume a common single analytic identical beam model for the antennas in the array. In the case of phased arrays this was never going to be the case anyway.

There are multiple causes which can result in poor dynamic range and thus limited imaging quality. The following list names several of them, however, does not claim to be complete:

- Pointing errors
- Insufficient knowledge of antenna beams
- Polarisation leakage and its time-variability
- Ionospheric non-isoplanatism and decorrelation
- Tropospheric decorrelation
- RFI and other non-linear effects
- Baseline-based non-closing errors
- Source variability
- Limitation of calibration and deconvolution algorithms especially for extended field and sources

- Inadequate software implementation

Many of the listed points are direction dependent and/or time-variable. In the case of direction dependent effects different sources in the field experience different effects. However, such effects might even be time-dependent as well, e.g. ionospheric changes, and thus limit the integration time or have to be incorporated into the measurement equation as a time-averaged effect which though is likely to lead to degradation of the final data product.

The paper is arranged as follows: Section 2 describes the theory of large interferometer telescopes and the measurement equation which can be constructed to represent/give an accurate description of the measurement process. Section 3 introduces standard calibration techniques as they are widely used in radio astronomy. In section 4 issues concerning and limiting the quality of polarimetric observations carried out by the new generation of instruments are discussed. Moreover, software packages and their suitability to calibrate future data are looked at and recent advances are described in this section as well. Section 6 focuses on issues which need to be solved and tries to establish what future work needs to be done in order to construct calibration pipelines for the new instruments. We conclude in section 7.

2 Fundamental theory

In the following we describe the measurement equation in radio interferometry. While the measurement equation has been widely used in research at optical wavelengths, Hamaker et al. 1996 were the first two emphasize its importance to radio astronomy. The measurement equation based on the Jones and Mueller matrix formalism represents a complete and mathematically elegant framework for the description of all kinds of full polarisation signal propagation effects.

2.1 Radio interferometry

An interferometer consists of a number antennas whose output signals get correlated. In this way aperture synthesis measures the spatial Fourier transform of the sky image. The technique is in our days widely used in radio astronomy.

The aperture plane of the interferometer is the plane perpendicular to the instantaneous direction from the array to a phase-reference center on the sky s_0. A baseline b_{ij} is defined as the three-dimensional vector between the locations of a pair of antennas i and j, projected onto this aperture plane. The components of b_{ij} are commonly measured in units of wavelength.

A so-called visibility is defined as the time averaged cross correlation product of the total electric field measured at two antennas, i and j, of the array with a time delay between the measurements, and is given for example for a single polarisation by:

$$V(\mathbf{b}_{ij}) = \int \langle |\mathbf{e}_p(\mathbf{s},t)| \cdot |\mathbf{e}_p(\mathbf{s},t - \mathbf{b}\cdot\mathbf{s}/c)| \exp(-2\pi i \mathbf{b}\cdot\mathbf{s}) d\Omega \tag{2}$$

where $\mathbf{s}$ is a location on the sky with respect to the phase reference centre, $|\mathbf{e}_p(\mathbf{s},t)|$ is the complex amplitude of the radiation emitted from location $\mathbf{s}$, $\mathbf{b}\cdot\mathbf{s}/c$ is the time delay of the signal arriving at the antenna with the longer light-travel path. The integration runs over the solid angle covered by the primary beams of the antennas. In order to ensure that time-averaged term in equation 2 gives the source brightness distribution, the maximum delay time has to be smaller than the coherence time of the signal. The visibility measured by an interferometer of baseline $\mathbf{b} \equiv (u,v,w)$ is then given by:

$$V(u,v,w) = \int \frac{A(l,m,n)I(l,m,n)}{\sqrt{1-l^2-m^2}} \exp(-i2\pi(lu+mv+nw))dl\,dm, \tag{3}$$

where $A(l,m,n)$ is the amplitude response of the interferometric array, $I(l,m,n)$ is the sky brightness distribution and $l,m,n = \sqrt{1-l^2-m^2}$ are the direction cosines (see e.g. Thomson et al. 1986). u, v, w are the components of b_{ij} where u and v are two-dimensional spatial frequencies and w describes the height of an antenna relative to the plane of the array in the direction of the phase reference centre on the sky, s_0. When $I'(l,m,n) = \frac{A(l,m,n)I(l,m,n)}{\sqrt{1-l^2-m^2}}$, the above equation becomes:

$$V(u,v,w) = \int I'(l,m,n) \exp(-i2\pi(lu+mv+w(\sqrt{1-l^2-m^2}-1))dl\,dm, \tag{4}$$

where $I'(l,m,n)$ is the modified brightness distribution. For planar arrays ($w \approx 0$) this relationship can be simplified to a two-dimensional Fourier transform. This is the Van Cittert Zernike theorem and forms the basis for interferometric imaging. Thus, as one measures the Fourier components of the sky brightness distribution, this method is also referred to as indirect imaging or Fourier synthesis. Until to date most operational interferometric telecope array facilities are well described by the planar form and thus the w-term is not an issue. However, this will not be anymore the case in the advent of large aperture arrays with baselines of several hundreds of kilometers.

2.2 The measurement equation

Electro-magnetic radiation can be described by two complex(-valued) components of the transverse electric field vector, $\mathbf{e}(t) = (e^p(t), e^q(t))$ incident to an orthogonal pair of feeds ensuring full polarisation. In the following we label the orthogonal polarisations as p and q. The measurable properties of $\mathbf{e}$ are given by the coherency matrix, $\rho = \langle \mathbf{e}(t) \otimes \mathbf{e}^\dagger(t) \rangle$ (outer product), where the angle brackets denote time averaging and $\mathbf{e}^\dagger$ is the Hermitian conjugate of $\mathbf{e}$. Note that four cross-correlation pairs are formed per baseline. The coherency matrix relates to the linear combination of Hermitian basis matrices:

$$\rho = \frac{1}{2} \sum_{k=0}^{3} S_k \sigma_k, \tag{5}$$

where σ_k, $k \in [0, 1, 2, 3]$ are the Pauli matrices:

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{6}$$

$$\sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_3 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \tag{7}$$

$$\tag{8}$$

and S_k are the Stokes polarisation vector components, $\mathbf{S} = (I, Q, U, V)$. Vice-versa the Stokes parameters can be expressed in terms of the coherency matrix: $S_k = \mathrm{tr}(\sigma_k \rho)$, where $\mathrm{tr}(A)$ is the trace of matrix A.

Jones matrices describe the modification to the in-comming radiation by propagation effects and receiver electronics. A Jones matrix is a 2-by-2 matrix of complex numbers that describes the action of an optical system on the x- and y-components of the electric field of an incident plane wave:

$$\mathbf{J} = \begin{bmatrix} J^{pp} & -J^{pq} \\ J^{qp} & J^{qq} \end{bmatrix}. \tag{9}$$

Hence, the output of a system is given by the convolution: $\mathbf{e}' = \mathbf{J}\mathbf{e}$. The Jones matrix corresponding to ionospheric Faraday rotation is e.g. given by:

$$J_{\mathrm{rot}} = \begin{bmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{bmatrix}, \tag{10}$$

where $\gamma = \mathrm{RM}/\nu^2$.

This can be converted to a so-called Mueller matrix:

$$M_{ij} = \frac{1}{2}\mathrm{tr}(\sigma_i \mathbf{J} \sigma_j \mathbf{J}^\dagger), \tag{11}$$

where σ_i are again the Pauli matrices. Jones matrices are handled multiplicatively, so that the total Jones matrices of direction dependent and independent effects is given by the (usually non-commutational) product of individual Jones terms of specific effects. The 4-by-4 Mueller matrix is therefore the outer product of the two entire/complete antenna based Jones matrices, $\mathbf{M}_{ij} = \mathbf{J}_i \otimes \mathbf{J}_j^\dagger$ (with i and j here labelling antennas of baseline pairs). Mueller matrices are as Jones matrices multiplicative.

Following Hamaker, Bregmann & Sault 1996 the full polarimetric measurement equation describing astronomical imaging using interferometer telescopes can thus be for a single baseline, frequency channel and integration time step generally/generically expressed as:

$$\mathbf{V}_{ij}^{obs}(\nu,t) = M_{ij}^{vis}(\nu,t) W_{ij}(\nu,t) \int M_{ij}^{sky}(\mathbf{s},\nu,t) \mathbf{I}^{sky}(\mathbf{s},\nu,t) \exp\left(2\pi i \mathbf{s}\cdot\mathbf{b}_{ij}\right) d\mathbf{s}, \tag{12}$$

where $\mathbf{V}_{ij}^{obs}(\nu,t) = [V^{pp}\, V^{pq}\, V^{qp}\, V^{qq}]_{ij}^T$ is the visibility sample as observed by the two antennas denoted by sub-scripts i and j, which have a baseline separation given by the vector $\mathbf{b}_{ij}$. The visibility sample is/can be weighted by the measurement weight $W_{ij}(\nu,t)$ and $M_{ij}^{vis}(\nu,t)$ is the complex direction independent gain. $M_{ij}^{sky}(\mathbf{s},\nu,t)$ is the direction dependent gain and thus depends on the direction $\mathbf{s}$. $\mathbf{I}^{sky}(\mathbf{s}) = [I^{pp}\, I^{pq}\, I^{qp}\, I^{qq}]^T$ is the sky brightness in direction $\mathbf{s}$ and can most generally vary with time and frequency. Note that vectors $\mathbf{V}_{ij}^{obs}$ and $\mathbf{I}^{sky}$ are full polarisation vectors. M_{ij}^{vis} and M_{ij}^{sky} are the Mueller matrices and are obtained from the outer products of the directional independent and dependent Jones matrices. Note that for accurate polarisation calibration and high dynamic range imaging the off-diagonal terms in these Mueller matrices are non-negligible and a full polarisation calibration is necessary.

During an observation the *uv*-plane is filled with measurements as the baseline changes with respect to the phase centre due to earth rotation yielding a coverage of the *uv*-plane. Moreover, an interferometer of n_a antennas takes $\frac{1}{2}n_a(n_a-1)$ *uv*-samples (one for each possible baseline) at a time. Thereby it is important to have a high frequency and time resolution in order to avoid aliasing effects which are otherwise inevitably and irreversible. In order to include the coverage of the *uv*-plane, the measurement equation can be expressed in block matrix form as (see

Rau et al. 2009):

$$\mathbf{V}^{obs}_{cn\times 1} = [\mathbf{K^{vis}_{cn\times cn}}][\mathbf{S_{cn\times cm}}][\mathbf{F^{vis}_{cn\times cn}}][\mathbf{K^{sky}_{cm\times cm}}]\mathbf{V}^{sky}_{cn\times 1}, \tag{13}$$

where n is the number of visibilities, m the number of discrete spatial frequencies (pixels), c gives the measured correlation, $\mathbf{S_{cn\times cm}}$ is an projection operator mapping the image plane to the visibility samples and $\mathbf{F^{vis}_{cn\times cn}}$ the Fourier transform operator.

However, as discussed in section **??** this way of expressing the Measurement Equation is limited and it can be generalised/ported to a three-dimensional description.

2.3 Existing standard calibration methods

In order to make a corruption-free image, $\mathbf{V}^{obs}$ needs to be corrected for the effects of $\mathbf{M}^{vis}$ and $\mathbf{M}^{sky}$. Standard calibration techniques most often/commonly neglect direction dependent effects (see section 2 for a description of such). In the past such have been neglected since more stringent limitations were placed by other sources of error and (other) instrument properties. Moreover, also science goals at the time could be well achieved by the standard algorithms in the common observing wavebands. To achieve the science goals of future telescopes - for example to study cosmic magnetism or reionisation in the early Universe at very low frequencies - dynamic ranges, which are several orders of magnitude higher than the ones obtainable via standard methods, and thus far more elaborate calibration algorithms are required. However, the calibration techniques presented in this section are still widely used to date and have been successful applied to data obtained from instruments over the last decades.

In general the unknowns are the sky brightness (imaging) and the instrumental effects (calibration). The most straight forward way to calibrate is thus to observe a calibrator source which is in the most optimal case an unresolved point source with invariable/stable surface brightness. Such a calibrator source has to be observed during repeated cycles. The maximum duration between calibrator observations is commonly limited by the stabilities of LNAs (low noise amplifiers) and of the phase transfer systems of the antennas in an aperture array. In existing interferometers the time interval between repeated calibrator observations is usually of the duration length of approximately one hour during which the system can be regarded as adequately stable. However, at low radio frequencies changes in the ionosphere and other instrumental effects (most certainly) request shorter calibration cycles. Thus, this calibration strategy can result in significant overheads and might be not suitable for next generation instruments with high bandwidths and large numbers of channels.

Apart from calibration using calibrator sources SelfCal is another established calibration technique widely utilised in radio interferometry. The advent of (scalar) self-calibration or SelfCal in the 1980s marked a landmark in radio astronomy. Achievable dynamic ranges increased significantly by the introduction of the method. However, SelfCal is unable to deal with many direction-dependent effects, such as antenna pointing errors and ionospheric and tropospheric refraction.

Traditional SelfCal solves simultaneously for the source brightness and the complex-gain errors of the antennas. It is an iterative method which alternates between two steps. First it uses a recent estimate of the sky brightness to improve the current estimate of the antenna errors. This estimate can be obtained from a previous observation or the present data. The observed visibilities are then corrected for the current antenna errors to improve the sky brightness distribution and the algorithm is iteratively repeated. SelfCal in this form only works when it is applied to data taken by a homogeneous interferometric array. From past experience it has been found that the method converges and has been found to be quite robust in most practical situations. However, formally it is not possible to prove that the limit the algorithm converges to is a unique one. Certainly one possible limit the algorithm can converge to is the true antenna gains and sky brightness distribution. Furthermore, the overall brightness scale is left undefined by SelfCal. Note that SelfCal relies on two assumptions. The first one is that the "instrumental" effects are antenna based and secondly that the sky is almost empty with a non-zero source brightness only in a minor fraction of the sky. For SelfCal to work the observed data has to be well conditioned meaning that the taken number of visibility samples is (much) greater than the number of unknown antenna gains and non-zero source pixel brightness values.

SelfCal demonstrates well why imaging (knowledge of the sky brightness distribution) and calibration (correction of observing effects) are closely related and dependent on each other. The image and the artifacts by the observing process have to be constraint simultaneously to ensure best possible results.

However, the standard approaches are deemed to fail in applications to deep wide field observations. Calibration relies on a priori knowledge of the true sky emission or it being fairly simplistic which is very unlikely to be the case for future experiments. Especially in the case of directional dependent gains as the calibration varies over the field of view such information is unlikely to be available.

3 Observations: State of the art

In this section we briefly summarise the current state of practice in deep, wide-field low-frequency observations of radio interferometers which are at present in regular science operation.

In the past advances in dynamic range have been strongly correlated with the thermal sensitivity of the constructed arrays. Apart from a few deep observations, in most cases sensitivities of common contemporary arrays for reasonable observing times are not high enough to require very sophisticated calibration techniques which include directional-dependent or time-variable effects. Furthermore, traditional radio interferometer instruments have at most a few dozen of well-manufactured antenna and receiver units.

Interferometric low-frequency instruments in current and recent science operations are the Westerbork Synthesis Radio Telescope (WSRT), the Giant Metre-wave Radio Telescope (GMRT), the Very Large Array (VLA)[1] and Australia Telescope Compact Array (ACTA). Commonly these instruments can achieve dynamic ranges of ~ 10000 over most of their low-frequency range. Note that the science of future aperture arrays requires order of magnitudes higher dynamic ranges ($\sim 10^7$).

To the knowledge of the author the highest dynamic ranges achieved in specific observations so far are reported for the WSRT instrument (see e.g. de Bruyn 2006). The WSRT is an array in the Netherlands whose systematics have been intensively studied and are well known. The array is an equatorially mounted east-west array that can make use of redundant baselines for calibration purposes. With 14 dishes the array is a typical classic radio telescope which usually have at most a few tens of receivers. Net polarisation dynamic ranges have been achieved of up to $1-2\times10^6$ for observations at 21 cm. In the case of the WSRT, a purpose-built analysis software package called NEWSTAR which removes direction-dependent effects by peeling (see section 4.3) has been employed in order to achieve such high dynamic range.

[1]Due to upgrade to EVLA actually the array is at present not operational anymore.

4 Calibration and Imaging for wide field polarisation observations

Calibration is defined as an algorithm applied to observations that makes the data free of corruptions caused by the observing process. This is commonly done by solving for free parameters of a model which describes all relevant parts of the observing process. In case that one has insufficient knowledge about the sky brightness distribution, this model apparently has to also include a parametrisation of the true sky apart from modelling the observing process. Based on the assumptions and parametrisations one derives a measurement equation for the observed data. Obviously in the case that an invalid a priori model and parametrisation are assumed, the calibration fails.

In particular, in this section we discuss effects which are likely to cause significant complications for polarisation data calibration of future interferometers and describe recent devised algorithmic approaches to correct for them. While in standard calibration, which often makes use of calibrator sources, the sky is considered to be well-known, modern calibration techniques will have to include parameterisation of all parts of the measurement equation, such as the directional dependent terms, $\mathbf{M}^{\text{sky}}$, and to some extent the sky brightness. Simplifying assumptions are likely to bias the calibration. At the sensitivity and resolution levels of the new instruments, the sky is expected to appear more complex than it is the case for limits of operational arrays. However, degeneracies might occur between sky and calibration parameterisation and it might not be possible to always find unique solutions to which the calibration converges. Therefore, in the following imaging and deconvolution techniques are discussed as well which are utilised to improve the estimate of $\mathbf{I}^{\text{sky}}$, the so-called reference sky (see also section 4.4). An inaccurate/incorrect reference sky most certainly affects the calibration terms, $\mathbf{M}^{\text{sky}}$ and $\mathbf{M}^{\text{vis}}$.

4.1 The *w*-term, a geometric effect

In the case of large fields-of-view which is typically the case at low radio frequencies the integral given in equation **??** cannot be reduced anymore to a simple two-dimensional Fourier transform. Using such a simplified analysis inevitably leads to distortions away from the chosen phase centre. Obviously, the further the direction $\mathbf{s}$ is off-setted from the phase centre, the larger is the distortion. The magnitude of the apparent distortion for a co-planar array in dependence on the source azimuth is given approximately by:

$$\Delta\theta \approx \frac{\theta^2}{4.125x10^5} \sin z, \tag{14}$$

where z is the instrumental zenith angle. For non-co-planar arrays the situation is more complicated than the simple shift of position that occurs in the case of the co-planar. In any case the the point spread function with which the sky image is convolved is not anymore position-invariant and standard deconvolution algorithms are unsuitable.

Several algorithms have been developed to correct for the distortion caused by the w-term. A way to deal with this effect is faceting which can be applied in the image as well as in the aperture domain (see e.g. Sault et al. 1999, Cornwell & Perley 1999). As indicated by the name the method divides the field of view into a number of facets. Faceting yields undistorted sky images with an approximate space invariant point spread function. Faceting thus is advantages for succeeding deconvolution, especially/particularly if the emission is extended. The computational cost of the different faceting algorithms is similar.

Another way to correct for the distortion caused by the w-term is the w-projection algorithm which is based on absorbing the expression $\exp(w\sqrt{1-l^2-m^2}-1$ into the direction dependent gain, $\mathbf{M}^{\mathrm{sky}}$ (see equation 12). This way one corrects for the w-term during image deconvolution. A recent detailed description of the algorithm is given in Cornwell et al. 2008. It is reported in recent literature (see e.g. Rau et al. 2009) that implementations of the w-projection method are in practice about an order of magnitude faster than faceting algorithms. An algorithm closely related to w-projection is the w-stacking. Implementations of w-stacking have been developed by Cornwell et al. and will be incorporated in the ASKAP software package (ASKAPSoft) which is not yet available.

4.2 Directional dependent and time-variable effects

The in section 2.3 presented standard calibration techniques (on-source calibration and SelfCal) neglect time-variable and direction dependent effects. These however are crucial to make use of the wide fields-of-view and bandwidths of future radio interferometers.

Directional dependent effects are due to antenna illumination patterns, parallactic angle and ionospheric and tropospheric effects.

4.2.1 Primary beam

Aperture arrays and dishes with focal plane arrays have primary beams that are less stable than the ones of single pixel primary feeds. In any case the primary beams vary with frequency. The illumination pattern on the sky changes in case they are not co-rotated and generally due to receiver mounts and geometric distortions the beams are azimuthally asymmetric. Moreover, beam patterns are likely to differ for different observing units, i.e. aperture array stations or dish antennas with single pixel or focal plane array feeds. Obviously, such beam pattern asymmetries and variations have a negative effect on the achievable dynamic range and image quality and make calibration for these beams a real challenge. Especially the polarisation purity can be affected by these primary beam variations and unknowns. Calibration and imaging have to take the full polarisation of the signal into account. The primary beam of an instrument introduces instrumental polarisation due to the reception properties of the feeds. In the case that the antennas and thus their receiver feeds do not track the rotation of the sky, as it is characteristic for any radio telescope without an equatorial mount, this gives rise to another cause of variation of the the instrumental polarisation over the observation duration. Also antenna pointing errors result in different direction dependent Mueller matrices (see equation 12) for each baseline. All these effects require calibration and correction with high accuracy. Usually this can be done by solving an adequately parametrised measurement equation.

4.2.2 The Ionosphere (and atmospheric disturbances)

Corruptions caused by the ionosphere are one of the severely limiting factors for highly accurate polarisation calibration. The ionosphere affects the phase in a spatially and time varying manner across the antenna beam. Due to the extent/thickness of the ionosphere it is usually for the new generation of low frequency experiments not sufficient to approximate it to be a single layer flat phase screen. Knowledge of the three-dimensional electron distribution is necessary.

Moreover, the new generation of telescopes will consist of a large number of stations or respectively dishes over a long range of baselines. While the fields of view of stations or dishes close to each other experience the same or at least a similar ionospheric phase screen, stations of long baselines are certainly looking through different parts of the ionosphere. This can cause signal decorrelation. Moreover, similar decorrelation effects can be also caused by the troposphere. Variations in the ionospheric phase screen happen on the time scale of minutes which thus places an upper limit on the longest integration times between successive calibrations. Via field-based calibration which measures the shift of compact sources (see Cotton et al. 2004) local ionospheric gradients can be estimated and the entire field can be corrected by a polynomial fit to the phase estimates. However, the method is limited to baselines of a few kilometres and fields of view of several square degrees and is not applicable in the case of strong ionospheric refractive disturbances. Another method to calibrate for ionospheric effects is presented/discussed in the next section.

In the case of ionospheric disturbances, apart from affecting data as a complex phase screen, the ionosphere causes/introduces further complications in the analysis of the data and thus its calibration due to frequency-dependent Faraday rotation of the polarisation angle.

4.3 Peeling

In this technique the complex gain is estimated towards sources across the field-of-view. The method relies strongly on prior knowledge of the sources which has to be obtained from earlier observations or possibly in an iterative approach from earlier steps in the calibration. Therefore, one has to rely on having a good sky model at hand. The sky model is usually derived by deconvolution techniques as they are, for example, described in the section below (see section 4.4.1). The method has been successfully applied to data of conventional radio telescope arrays, such as the WSRT (see section 4). The method is an iterative one. After removal of the a priori known sources from the visibilities:

$$\mathbf{V}_{ij}^{corr} = \mathbf{V}_{ij}^{obs} - \sum_k \mathbf{J}_{ij}^{peel} \mathbf{V}_{ij}^{k^{model}}, \tag{15}$$

one proceeds by applying the technique to the brightest sources in the residual image. k denotes all sources to which the peeling solution is applicable. Note that the method (is possibly) deemed to break down in very wide field aperture array observations (at low frequency bands) which might contain complex emission structures on a number of scales. Nevertheless, there are efforts under way to extend the method to such kind of wide field complex data sets (see Nijboer & Noordam 2007). Mitchell et al. 2008 investigate a real-time peeling implementation for the Murchison Widefield Array.

4.4 Reference sky model

Many of the recently proposed strategies incorporate the idea of a reference sky model or at least some a priori knowledge of the sky brightness. Therefore in the following section we describe image deconvolution algorithms which can be of help to derive such. It is absolutely necessary that any used prior information is reliable and the sky reference model is time-independent. However, how much current observations and conventional techniques are of any help to achieve a reliable sky reference model is questionable. Due to the (very) long baselines which the new instruments possess, many sources which appear point-like to existing instruments, will be resolved. Thus they cannot be treated as point sources anymore, but have to be modelled as extended sources using, e.g., shapelets (see e.g. the contribution of de Bruyn et al. to the SKA2010 in Manchester). Moreover, the new facilities will have wide frequency bands and the source structure is likely to change over the observing band. For these reasons the source models will have to be far more complicated than currently generally assumed. Moreover, sources might even exhibit/show time-variable behaviour at non-negligible levels. At the same time, due to the increased sensitivity, many more sources will be detected and will have to be processed. This will not only affect the calibration of the instruments, but also the imaging and deconvolution. Because of their high sensitivity the new instruments are capable of detecting very weak sources, but they will have to do so in the presence of all the strong sources already known. Note that strong sources which are not even in the main field of view, can affect the observation through side lobes of the primary beam. Some iterative deconvolution and/or peeling applications might help with these issues in case the instrument beams are well known.

4.4.1 CLEANing algorithms

Most ideally one derives a sufficient calibration by solving an adequately parameterised measurement equation for best-fit values of its calibration parameters. However, this is in reality due to a lack of prior knowledge about the sky as well as the measurement process itself and limited computing resources often infeasible. In order to obtain information about the true sky brightness distribution it is often necessary to invoke iterative deconvolution techniques and iterate between calibration and deconvolution steps.

Several deconvolution algorithms have been developed in the last three decades. The first ones, such as Hoegbom CLEAN and the Maximum Entropy Method, neglect correlation between different resolution entities (pixels) of the map and are not optimal for extended brightness structures. However, various types of CLEANing algorithms have been introduced since the time Hoegbom established the fundamental principle. Some of these incorporate scale-sensitivity. One such CLEAN algorithm is Multi-Scale (MS) CLEAN. As the algorithm works with a set of pre-defined scale sizes, its performance and computational cost depend very much on the choice of scales. In the case of a very non-optimal choice of scales, it experiences problems. Moreover, it ignores coupling between the set scales.

Another scale-sensitive deconvolution algorithm is ASP-CLEAN. This method utilises a parametrisation of the sky brightness in a collection of Gaussians and adaptively determines the local scale and position of the components in the map in a constrained optimisation of their parameters. Apart from the inclusion of scale-sensitivity, MS-CLEAN and ASP-CLEAN have been shown to converge generally in far fewer iterations than it is the case for the traditional Hogboem CLEAN. Drawbacks of these algorithms are that they are not linear and in some cases are challenging to automatise.

In the case of wide bandwidth and large channel number observations, the spectral dependence of the sky brightness has to be accounted for as well. A deconvolution algorithm that takes care of the sky spectral dependence and the spectral instrument response is Multi-Frequency (MF) CLEAN (see e.g. Sault & Wieringa 1994). The method describes (pixel) source spectra by a power law and a first order Taylor expansion. In its original form the MF CLEAN algorithm does not include spatial correlations between pixel sources. MF CLEAN can be combined with MS CLEAN, though, to obtain a scale- and frequency-sensitive deconvolution algorithm (see Rau et al. 2009).

Note that commonly these deconvolution algorithms are applied to a single correlation pair of feeds, e.g. pp, or Stokes parameter. With the assumption that different Stokes parameters are linearly independent, deconvolution cycles can be performed separately on the images of the Stokes parameters.

5 Software development

Several of the in the previous sections discussed algorithms have been implemented and included into principal software packages. There are two different approaches taken in the software development for the new generation of interferometers. Some software is specifically developed for particular telescopes to match the special needs of the instruments. The development of these packages often happens within collaborations and software products are often not yet publicly available. An example of such a package is ASKAPSoft which is built to suit the needs of the Australian SKA Pathfinder ASKAP, a dish array with focal plane array receiver units. Nevertheless, even these packages make often use of open standard routine libraries (, such as CASAcore). Other packages try to be multi-purpose applicable and are developed in an open source environment, e.g. CASA. However, these are not always optimal for every kind of telescope hardware. For example, aperture arrays, such as the LOw Frequency ARray (LOFAR), have certain calibration requirements which are sometimes not addressed by the multi-purpose packages. Thus, LOFAR is building its own automated polarisation calibration and imaging pipeline, which is taylored to instrumental, computing and scientific needs of the project and is thus purpose-built.

Table 1 lists some of the most widely used and newly up-coming software packages available in radio astronomy and their main features. Some of the packages have been developed for the recent operational telescope generation (AIPS and MIRIAD). The development of others (CASA, MeqTrees and ASKAPSoft) is driven to serve future arrays.

As required full Stokes polarisation calibration is part of all the new packages. However, the advancement and level of algorithmic implementation differ from package to package. The following list of algorithmic approaches states in which of the packages that are under development an implementation of the respective method exists:

- peeling: MeqTrees
- *w*-projection: CASA, ASKAPSoft (also *w*-stacking)
- faceting: CASA, ASKAPSoft, (AIPS)
- multi-frequency approaches: CASA (also including image cube based methods), ASKAPSoft, MeqTrees
- full Stokes measurement equation: CASA, ASKAPSoft, MeqTrees

For example, the available version of CASA has a so-called *polcal* routine which can determine instrumental leakage terms, so-called D-terms, utilising a (polarisation) calibrator source. However, the implemented routine is/seems to be still quite closely linked to standard polarisation calibration techniques which have been already utilised/implemented in the predecessor packages. Future observations might need to calibrate and image at the same time. CASA offers within its

CLEAN routine faceting and w-projection options. In CASA the w-projection and respectively faceting algorithms are controlled by the sub-parameters *wprojplanes* and *facets*.

A stringent requirement on all these packages is that they are able to handle, polarisation calibrate and image (full Stokes) next generation telescope data which will be taken at very high rates and will be of high complexity. Despite the recent developments to achieve all these goals will be a challenging task. To give an example, as can be seen in Table 1, (to the knowledge of the author) none of the listed packages is in fact yet entirely/fully ready to be run on multi-core multi-thread high performance computing architectures. A barrier/handicap seems often that most software development efforts suffer/lack strong personal/man-power support for implementation and testing purposes.

5.1 Recent results and applications

Most methods mentioned above have been implemented recently and are often still in a testing stage. For example, Smirnov et al. have included the correction of direction-dependent effects, such as atmospheric refraction and antenna beam uncertainties, into the MeqTrees package in a full measurement equation approach solving for such effects. For testing purposes they applied the software to WSRT data and achieved a dynamic range of $> 10^6$ over the entire field-of-view and obtained noise levels in the image as expected from the theoretical thermal noise limits (see Smirnov talk at SKA2010). There have been major efforts at the American National Radio Astronomy Observatory (NRAO) and the Australian CSIRO to implement many of the afore-mentioned algorithms into packages like CASA and ASKAPSoft. For example, the implementation of projection methods to correct for directional-dependent effects caused by antenna pointing errors and irregularities and changes in antenna voltage beam patterns are discussed in several papers (see e.g. Bhatnagar et al. 2004; Bhatnagar et al. 2006; Bhatnagar et al. 2008 and Cornwell et al. 2008). These authors often apply their implemented algorithms first to simulations (making use of the CASA simulation tools) and then also to archived VLA data to give proof of concept.

6 Where are we and what's next

Research in this area is ongoing. Although many ideas are being generated, only a limited number of new calibration approaches have actually been implemented

Software package:	MIRIAD	AIPS	MeqTrees	CASA	ASKAPSoft
Availability:	✔	✔	✔	✔	✘
Calibration:	✔	✔	✔(full Stokes)	✔(full Stokes)	✔(full Stokes)
Polarisation:	✔	✔	✔	✔	✔
Computing language:	Fortran77/C	Fortran66	C++/Python	C++/Python	C++/?
Parallelisation:	✘	✘	(✔)	Not Yet	?

Table 1: Available and up-coming software packages and their features.

and tested on real data. This is hardly surprising since only now the first of these new instruments are coming online and producing data which requires new algorithmic approaches. Hence, also the astronomical observer and software user community often still sticks to traditional methods and packages.

Processing data from the new instruments remains challenging and drives the research of signal processing in the area of low frequency radio astronomy. A number of algorithms which will help with polarisation calibration are suggested and partly implemented. However, many efforts are still at an early stage and require further testing and development. Plans exist for many up-coming and furure arrays to develop automated calibration and analysis pipelines so that the astronomical user will be supplied with the final scientifically useful data product. A major challenge is that algorithms have to be able to provide automated real time calibration due to the immense data intake and flow of the new generation of arrays. Due to huge data outputs it is infeasible to store the full polarisation *uv*-data for long times. Furthermore, a significant complication for these plans is that not only the numbers of array elements in comparison to classical radio interferometers will strongly increase, though, the envisaged technologies, such as aperture arrays and focal plane arrays, are often more complex and less stable than it is the case for the single pixel feeds of the currently still operational generation of instruments. In addition, the wide bandwidths of the new instruments will cause further complications due to the effects of the atmosphere at frequencies below $\sim$1 GHz and spectral variations of the sky over the observed frequency range.

Apart from the challenges described already throughout the text in detail, some further challenges which have not yet been widely discussed remain. Obviously in order to deal with data from the new instrument generation more complicated models of the sky and the observing process are needed, which in turn contain more unknowns that need to be extracted from the data itself. The increased station number will yield more baselines and thus equations - however, since this also introduces more complexity it might not be sufficient. It is thus important to find a suitable set of basis functions to model the time and frequency dependence of parameters since this will reduce the amount of unknowns that need fitting.

Another yet unmentioned important problem is radio frequency interference (RFI) mitigation. The radio frequency spectrum is rather crowded, and it is expected - even though most sites of new arrays are carefully chosen - that many observations will suffer contamination by different levels of RFI. Thus before a reliable calibration and imaging is at all feasible flagging and excision of the contaminated data is necessary. New fast algorithms have to be developed for these purposes as well. Further, it might be possible to use array signal processing techniques, such as null steering, to suppress the level of contamination of interference in the data. Nevertheless, also such techniques need to be implemented to respond in real time.

Moreover, the derivation of the Measurement Equation as presented in section 2 assumes paraxiality and is therefore only a valid approximation for narrow and at best medium wide field-of-views. While it may still hold for parabolic dish antennas of diameters $\gtrsim$ 10m at observing frequencies of several 100 MHz and above, it is unlikely that it does work for a dipole antenna which sees a major fraction of the sky. In order to generalise the standard Measurement equation formalism to arbitrary wide fields, one has to by deriving a van Cittert-Zernike relation that is valid over the entire celestial sphere and is fully polarimetric (see e.g. Carrozzi & Woan 2009). However, implementation of algorithms for full Stokes polarisation calibration are likely to be computationally very costly since many advantages of the form as presented in section 2, such as the use of the two-dimensional FFT, may drop away.

However the curse of dimensionality might well strike. Already equation **??** does not have a unique solution in the case one considers only a single baseline and limited knowledge about the sky signal since one has only four equations to solve for up to 10 unknown parameters. A way forward to avoid under-determined systems of equations is to use redundant baselines so that at least single antenna independent corruptions can be more easily solved for.

7 Conclusion

In this Memo we introduce and discuss contemporary and new calibration techniques. Recently, due to the advent of the new generation of radio telescopes, algorithmic and practical research in polarisation calibration and high dynamic range imaging methods has strongly increased. Several algorithms have been developed and implemented (see section 4) and some have been already made available in software packages (see section 5). However, most implementations and algorithms do not yet fulfill the requirements of being universally applicable and totally automated. Future facilities will require fully automated data processing, calibration and imaging. Highly accurate polarisation calibration is the fundamental requirement to achieve high dynamic range imaging.

An issue might be that some of the algorithms are developed with a certain hardware, e.g. dish antennas, in mind and might neglect needs of other antenna designs, such as aperture arrays. On the other hand, purpose-built software might be the only way to achieve all necessary requirements. The high data complexity of future arrays is accompanied by large data volumes. Thus, increases in algorithmic and computational performances are necessary. In order to reach ultimately the high aim of a black-box data analysis which relies only on a few parameters to be set by the user and presents the user at the end of an observing run with fully calibrated scientifically useful data, several barriers have still to be taken. Automatisation is a major one of them. Automatism and parallelisation of the algorithms is still at the onset of its development. Implemented algorithms need to be portable to high performance computing architectures. Even though manpower has been steadily increasing, these requirements will likely request further top-up of work force in the field.

Furthermore, to reduce pressure on the algorithm development, software design and computing requirements - as some of the instruments have not yet left their design stage, e.g.the SKA - it might be worth-while to investigate how to mitigate calibration errors by hardware design without increase of hardware cost. Finding suitable solutions/answers to all these challenges will be of critical importance for the success of the next generation of instruments.

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