Beam shapes for calibrating off-axis detections

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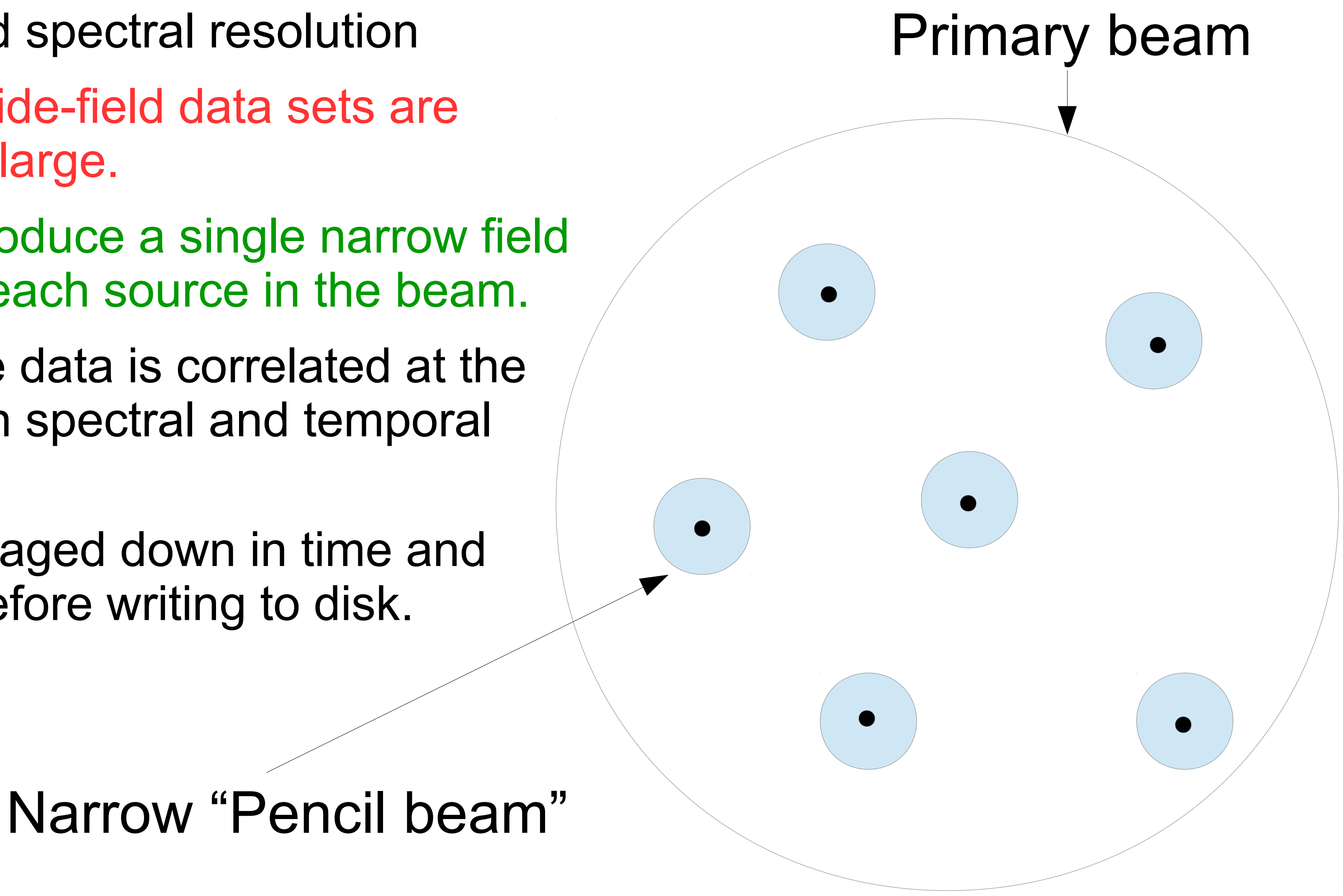
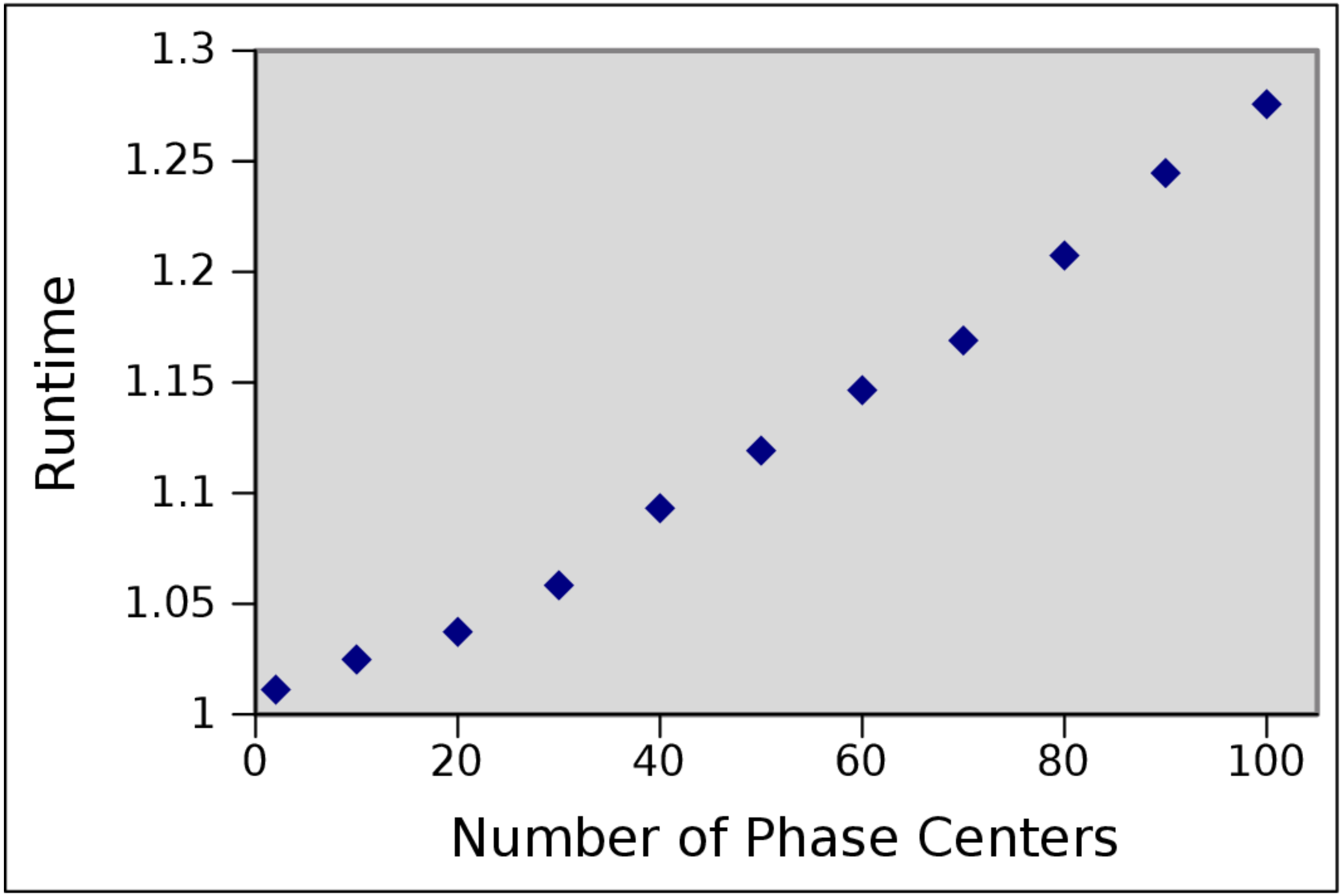
Multiple simultaneous phase centers

- An interferometer can map the entire primary antenna beams as long as the data is correlated with sufficient temporal and spectral resolution
- Problem : Wide-field data sets are prohibitively large.
- Solution : Produce a single narrow field data set for each source in the beam.
- Internally the data is correlated at the required high spectral and temporal resolution.
- Results averaged down in time and frequency before writing to disk.



Multiple simultaneous phase centers



Going from 2 to 100 sources requires only
30% more correlation time!

Airy Disk Beam Model

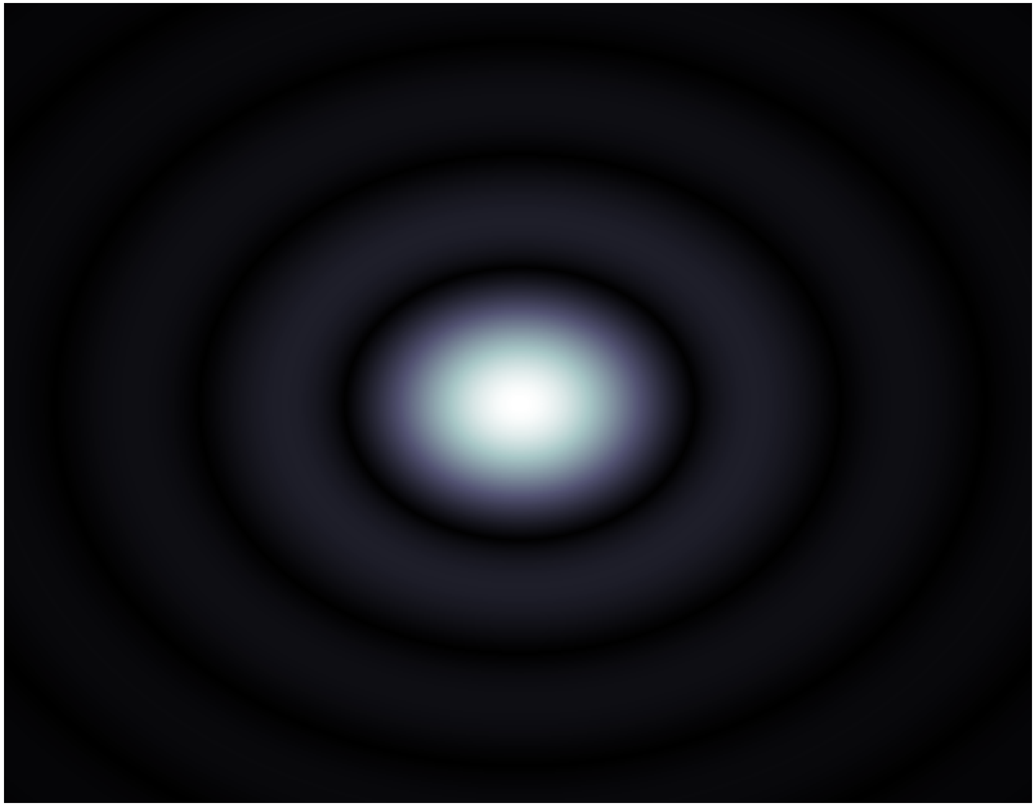
First order model : Uniformly illuminated circular aperture (Airy Disk)

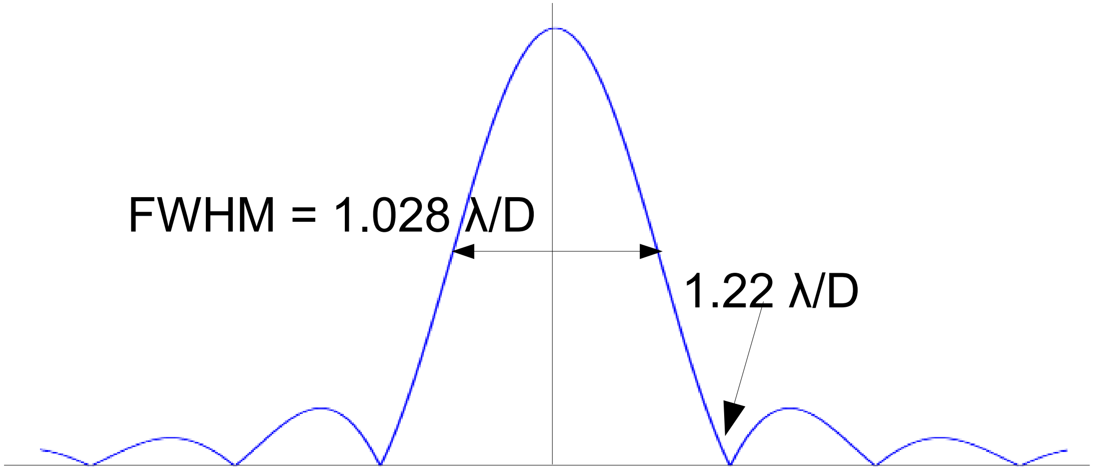
$$I(\theta) = \left| \frac{2J_1(z)}{z} \right|^2, \ z = \frac{\pi D}{\lambda} \sin(\theta)$$

D = Dish diameter,

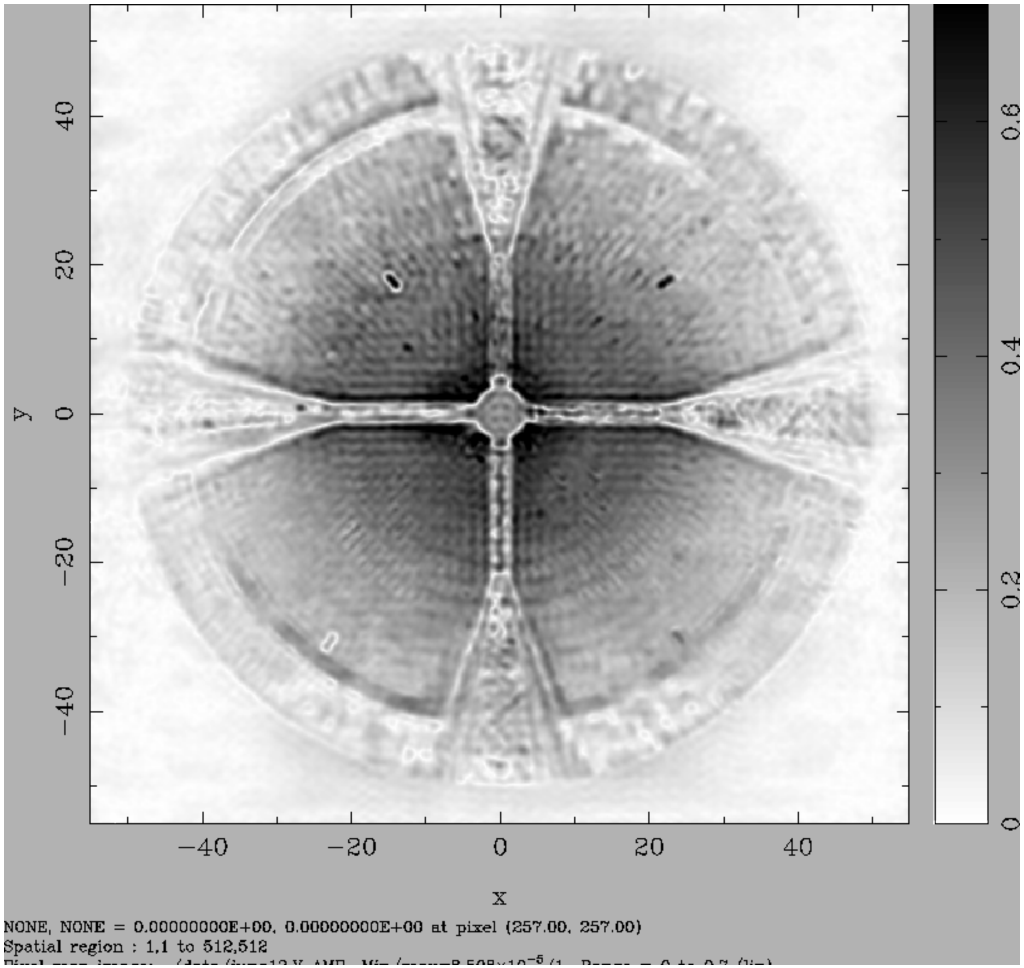
λ = wavelength

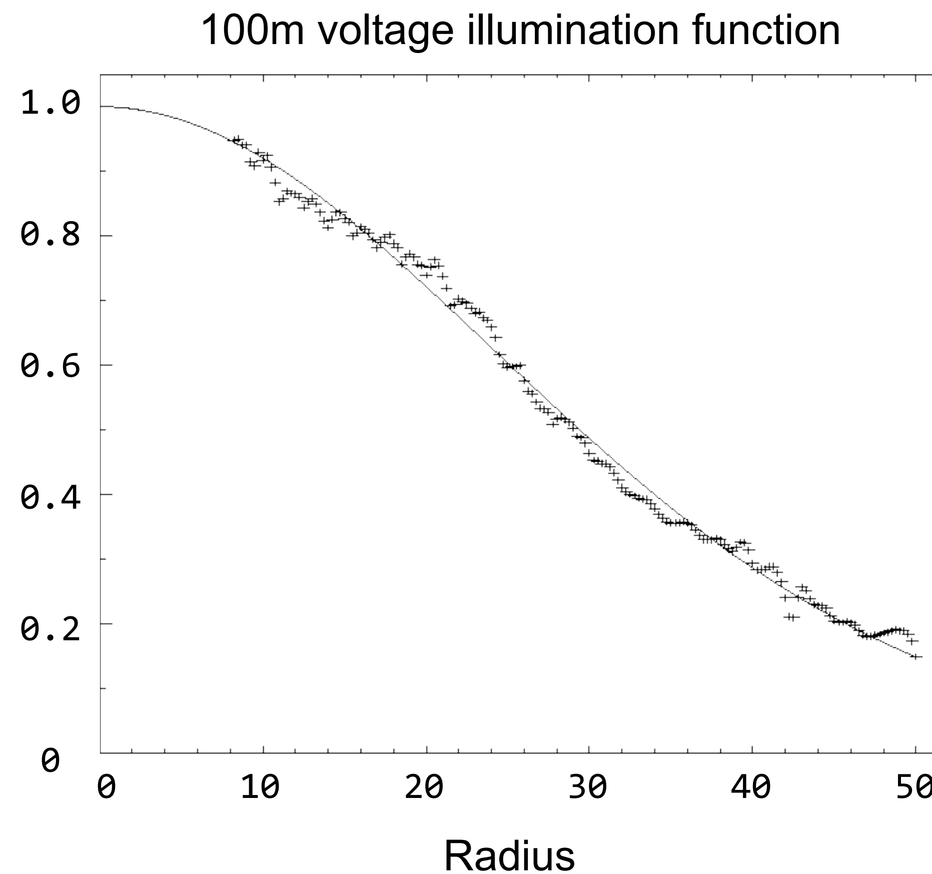
$J_1(z)$ = Bessel function of the first kind





Effelsberg illumination pattern @11.7 Ghz





The Effelsberg Holography Campaign - 2001

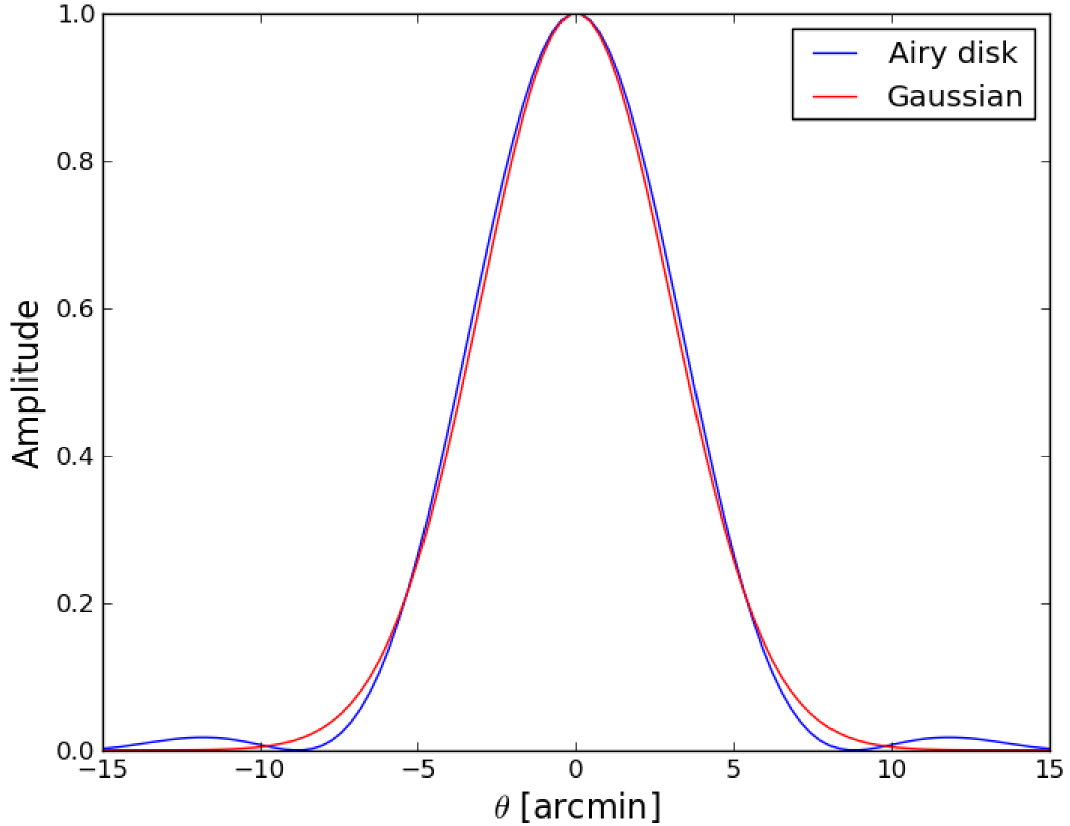
M.Kesteven, D.Graham, E.Fürst, O.Lochner & J.Neidhöfer

Gaussian model

- The Airy disk model can very closely be approximated by a Gaussian model

$$I(\theta) = A_0 e^{-\frac{\theta^2}{2\sigma^2}}$$

- The optimum fit is σ = 0.42λ/D, for apperture of width D



Model fitting beam shapes

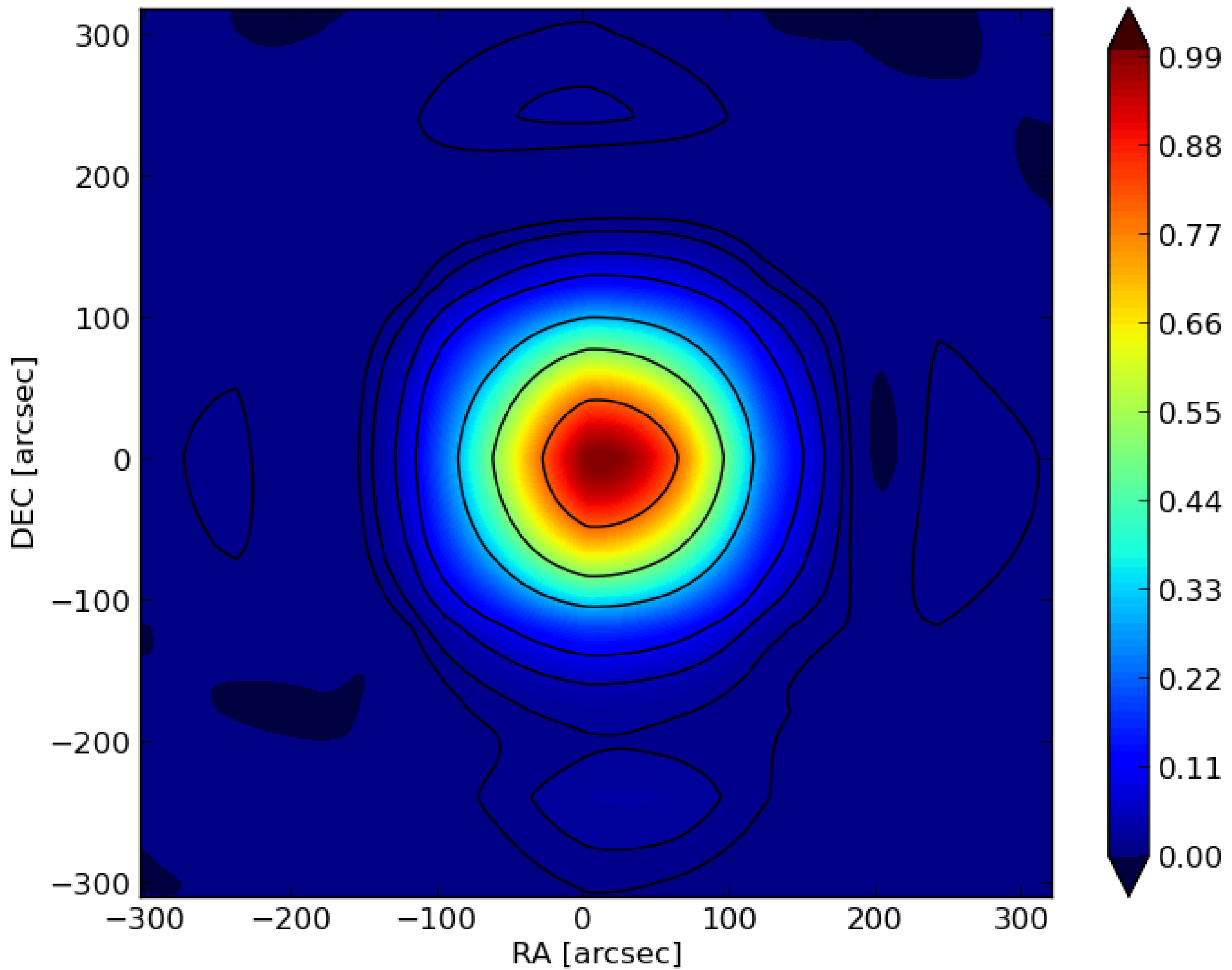
- Least-squares fitting : For a dataset **d**, containing N data points sampled at positions **r** = $r_0, r_1, \ldots, r_{N-1}$ and model given by the function f(**r**;**p**) find a set of parameters **p** that minimizes

$$X^2 = \frac{\sum_i \left| f(\boldsymbol{r}_i; \boldsymbol{p}) - d_i \right|^2}{\sigma^2}$$

- Python program was developed that fits a given beam map to a particular model using least squares fitting.
- Beam map is read from FITS file, other data formats can easily be supported. Can handle multiple input files.
- Supports Gaussian and Airy disk beam models for fitting. Very easy to add new beam models
- **Algorithm**

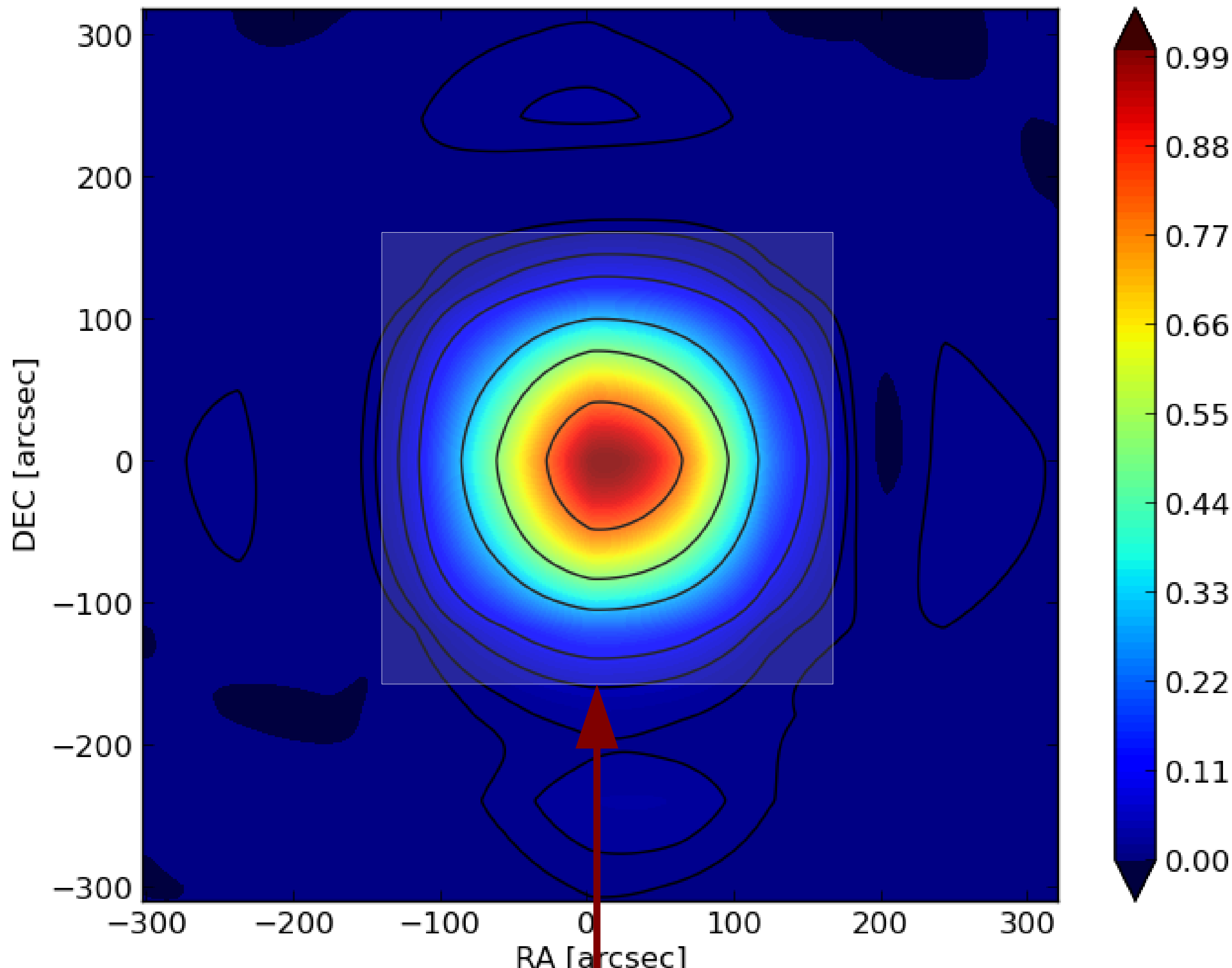
  1) Make initial estimate of model parameters
  2) Box data, restrict model fitting to data points inside beam
  3) Least squares fit beam model to data using the Levenberg–Marquardt algorithm.
  4) Reapply steps 2) and 3) using new model parameters

Effelsberg map of 3C48 @ 4.85GHz



Original map 750 x 450 points (not all shown)
2” resolution, 160 x 160 data points within beam.
Note the rectangular features at the primary beam edge

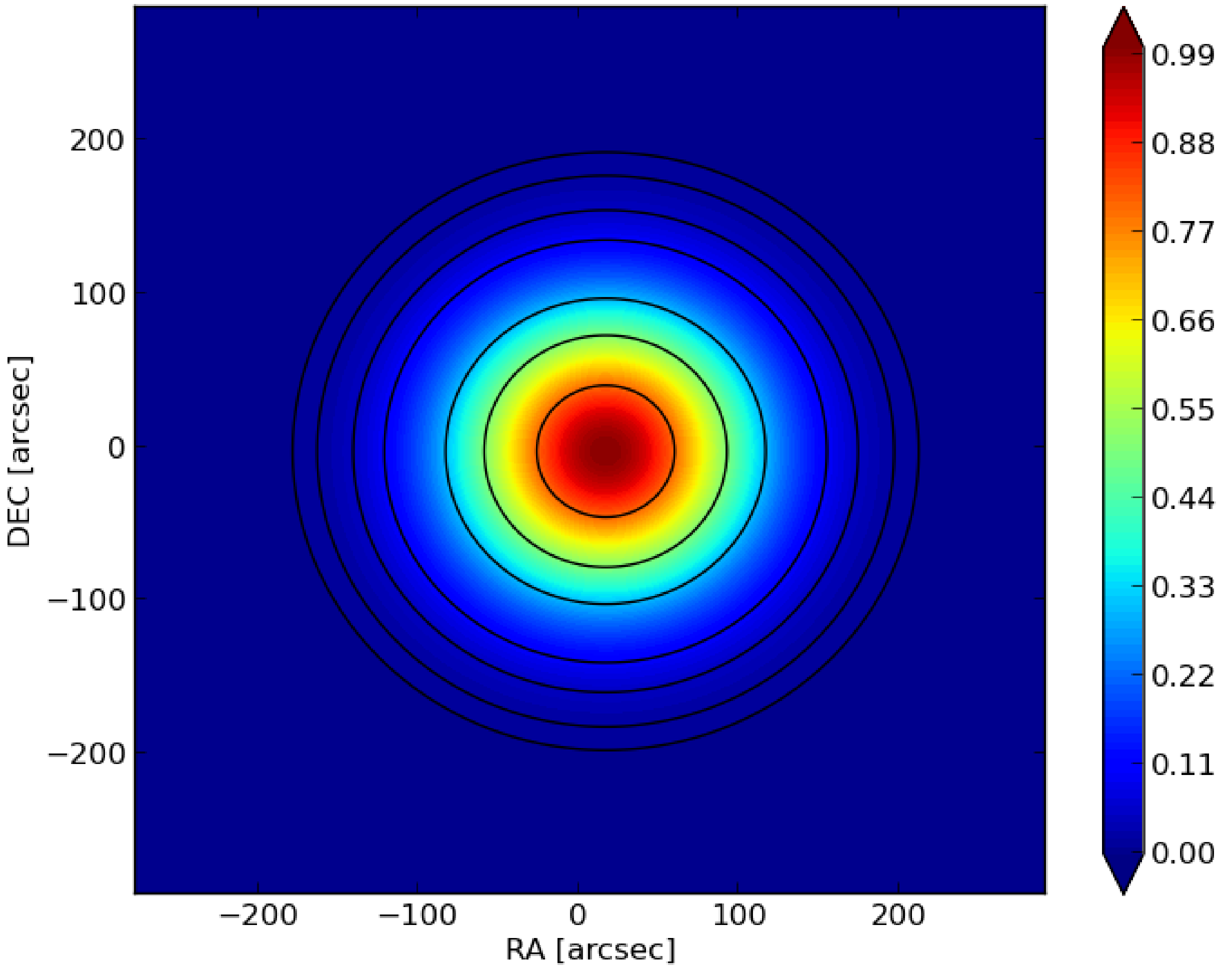
Effelsberg map of 3C48 @ 4.85GHz



Before model fitting we box the data, only these point are used in the fit

Gaussian model fit

Model : $I(x,y) = A_0 e^{-\frac{(x-x_0)^2+(y-y_0)^2}{2(z\lambda)^2}}$



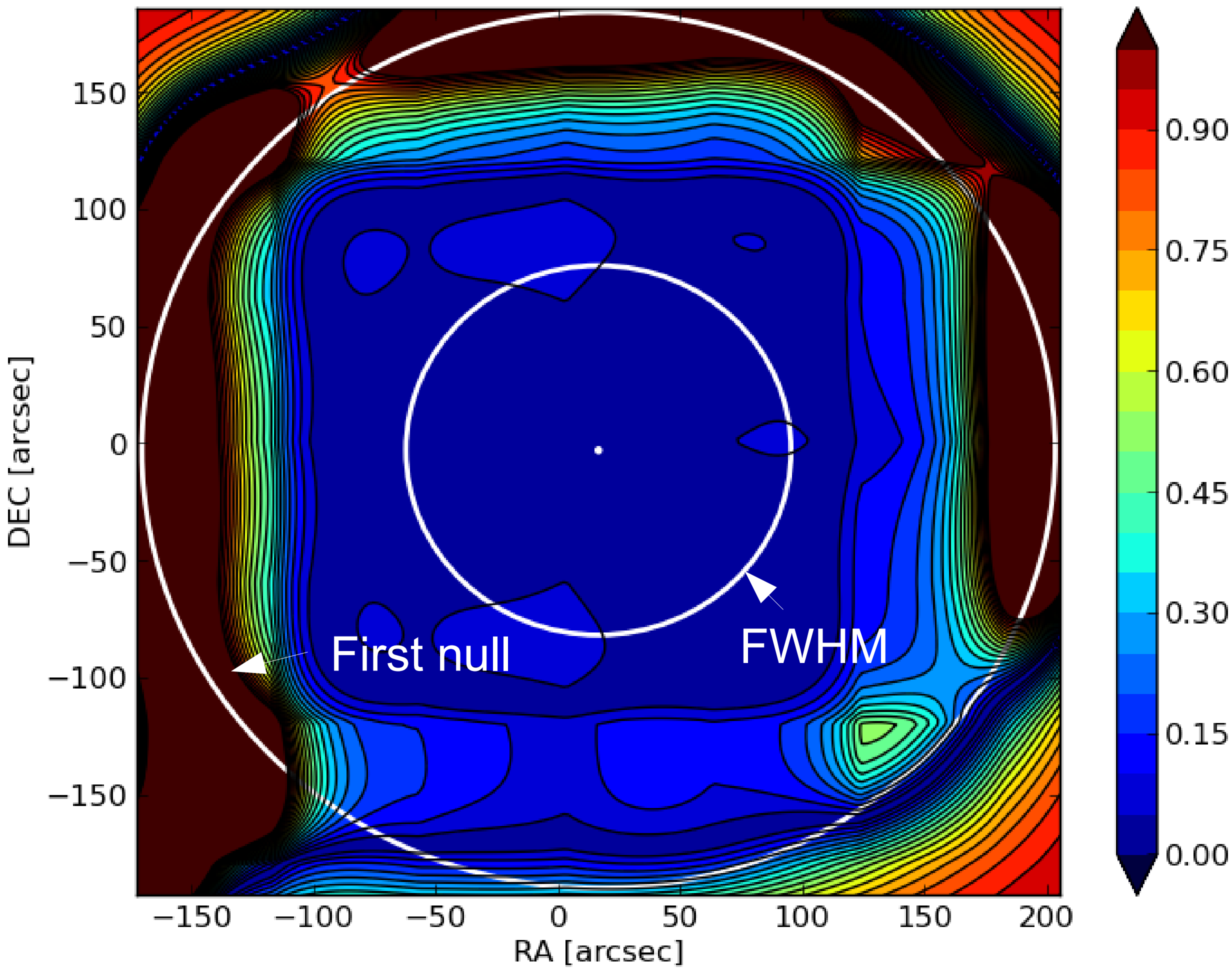
Results

```
A0 = 1.457e+04 ± 5.14e+00
X0 = 1.663e+01 ± 2.26e-02
y0 =-3.053e+00 ± 2.26e-02
Z  = 1.040e+03 ± 2.65e-01

FWHM = 151.6 arcseconds
```

Residuals Gaussian model

Model : $I(x,y) = A_0 e^{-\frac{(x-x_0)^2+(y-y_0)^2}{2(z\lambda)^2}}$



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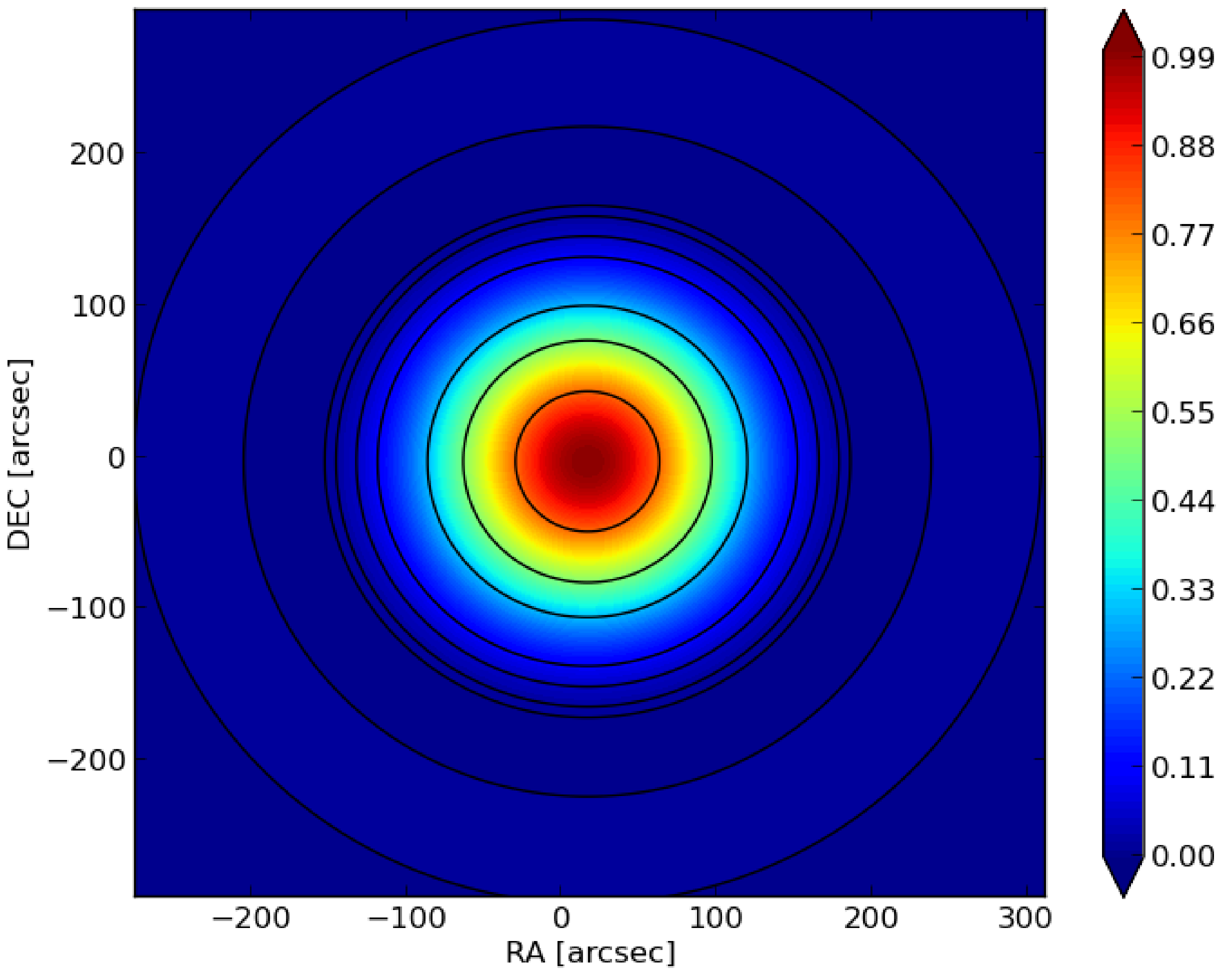
Z = 1.040e+03 ± 2.65e-01

FWHM = 151.6 arcseconds

Airy disk model fit

Model : $I(x,y)=A_0\left|\frac{2J_1(z)}{z}^2\right|, z=\sqrt{(x-x_0)^2+(y-y_0)^2}\frac{\pi D}{\lambda}$



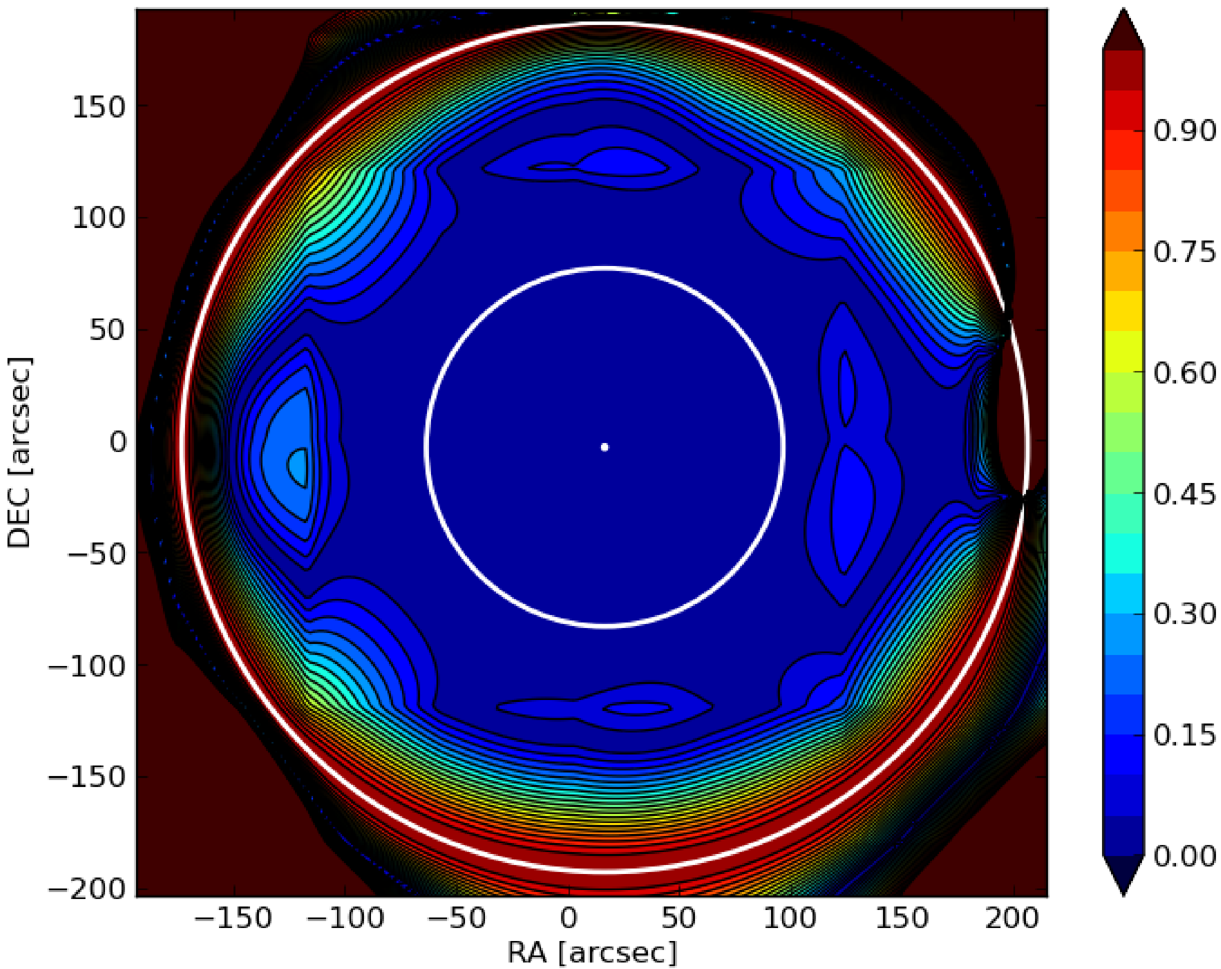
Results

```
A0 = 1.3974+04 ± 3.35e+00
x0 = 1.658e+01 ± 1.60e-02
y0 =-3.061e+00 ± 1.61e-02
D  = 8.200e+01 ± 1.32e-01

FWHM = 160 arcseconds
```

Residuals Airy disk model

Model : $I(x,y) = A_0 \left| \frac{2 J_1(z)}{z} \right|^2, z = \sqrt{(x - x_0)^2 + (y - y_0)^2} \frac{\pi D}{\lambda}$

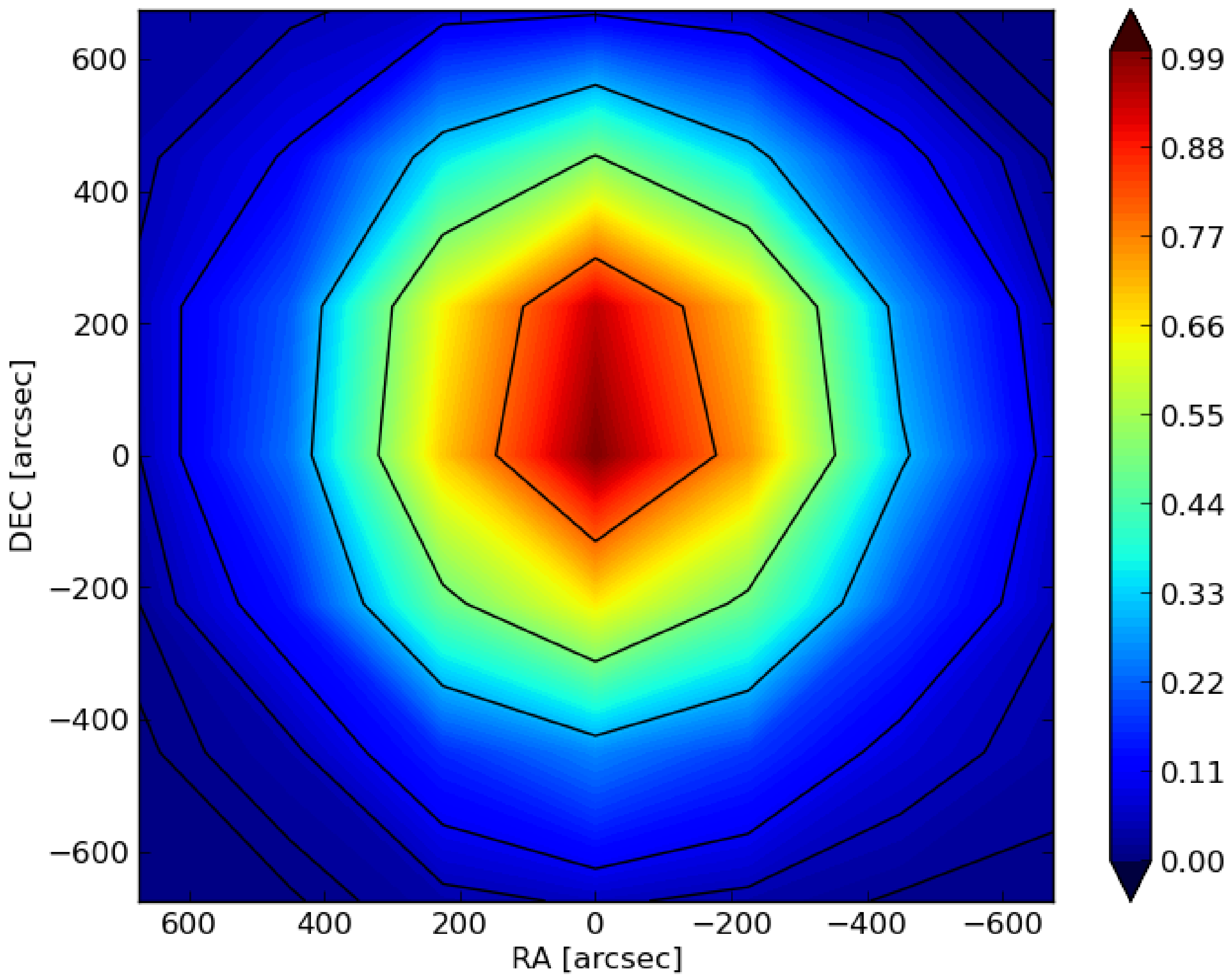


Results

```
A0 = 1.3974+04 ± 3.35e+00
x0 = 1.658e+01 ± 1.60e-02
y0 =-3.061e+00 ± 1.61e-02
D  = 8.200e+01 ± 1.32e-01

FWHM = 160 arcseconds
```

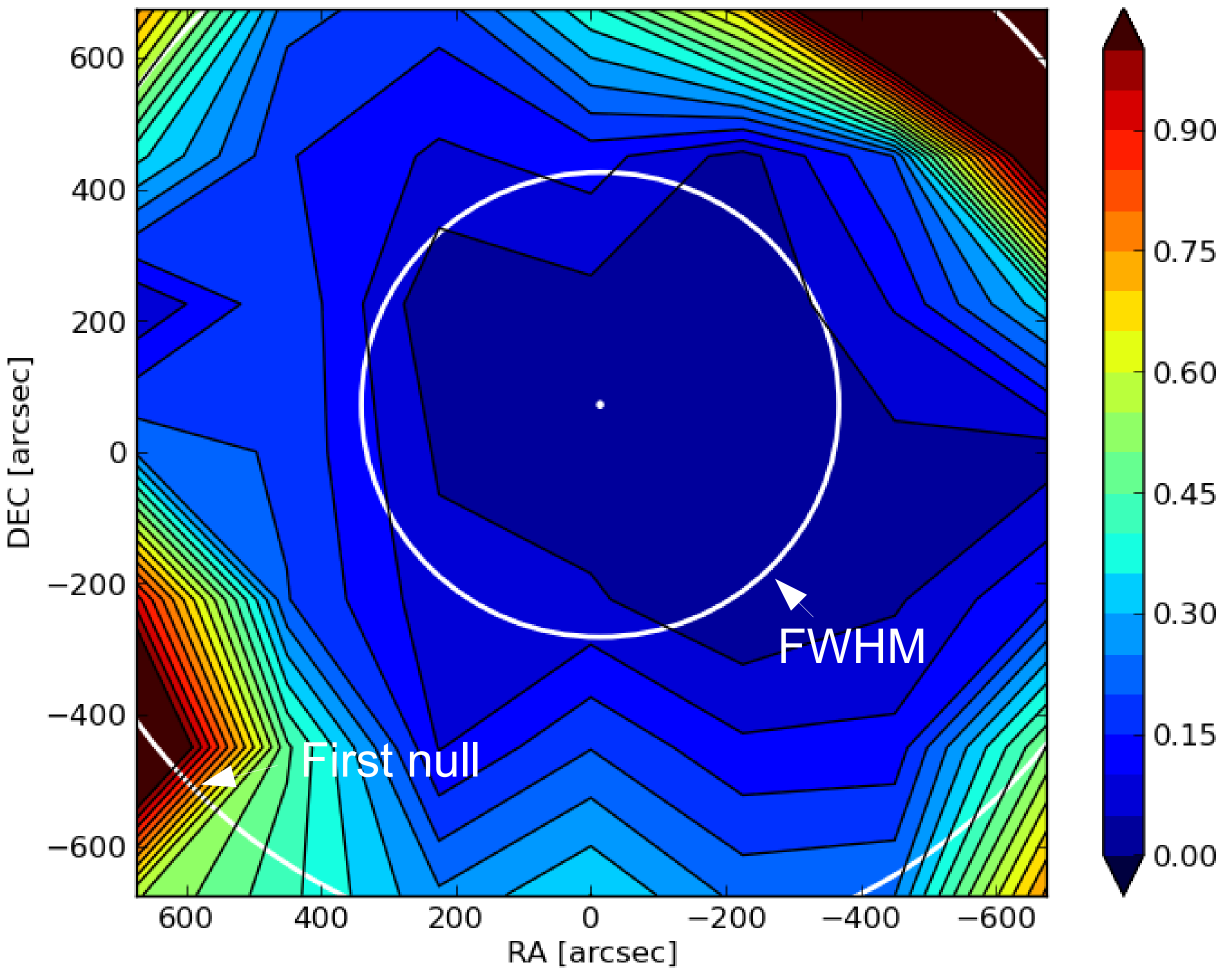
Onsala map of 3C286 @ 4.6GHz



Dataset contains 7 x 7 datapoints (3'40” resolution)
Each data point is a 800 point spectrum (3.2 MHz Bandwidth)

Residuals Gaussian model

Model : $I(x,y) = A_0 e^{-\frac{(x-x_0)^2+(y-y_0)^2}{2(z\lambda)^2}}$

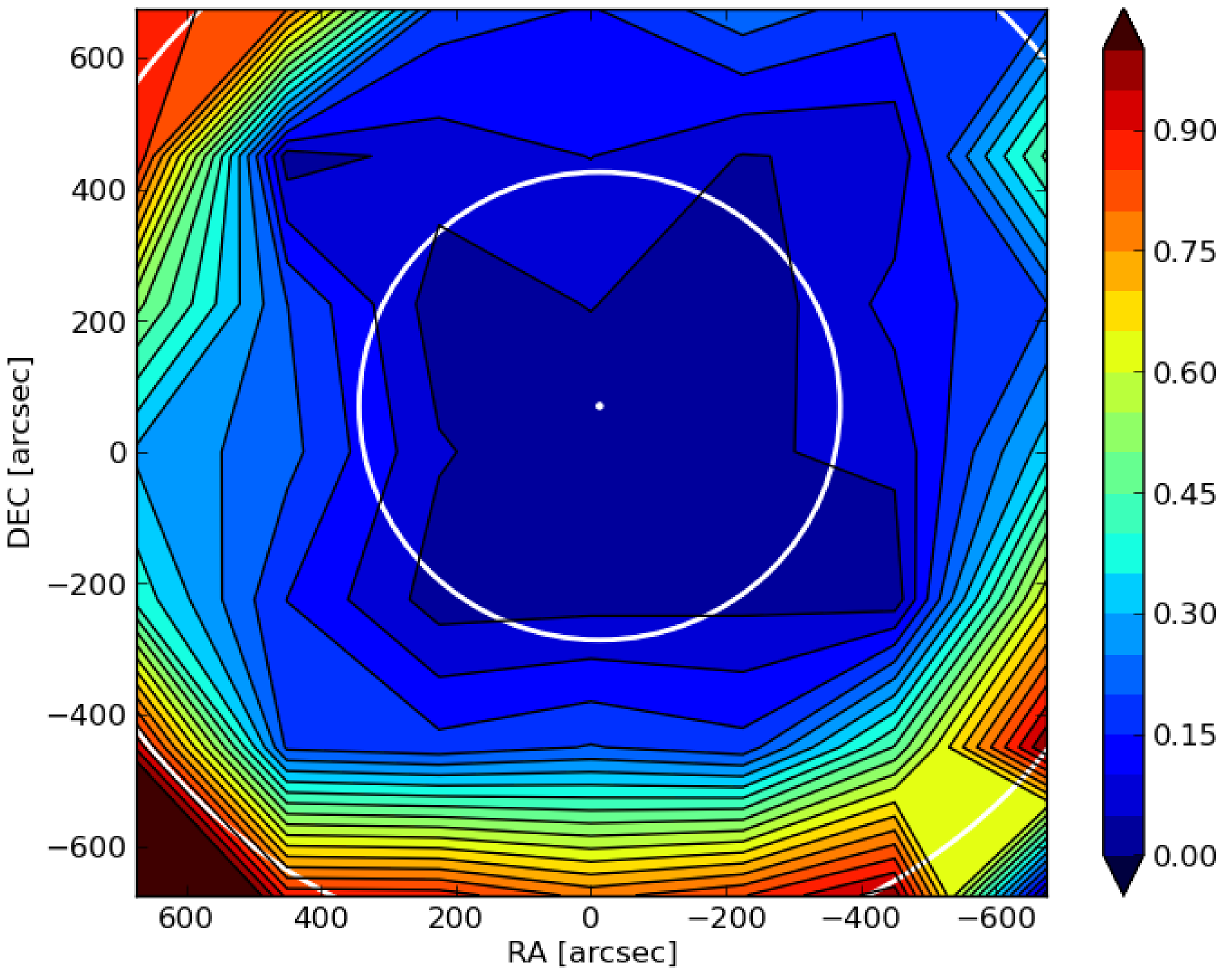


Results
```
A0 = 6.178e+01 ± 1.19e+00
x0 =-1.378e+01 ± 5.46e+00
y0 = 7.173e+01 ± 5.51e+00
Z  = 4.428e+03 ± 6.53e+01

FWHM = 680 ± 10 arcseconds
```

Residuals Airy disk model

Model : $I(x,y)=A_0\left|\frac{2J_1(z)}{z}\right|^2, z=\sqrt{(x-x_0)^2+(y-y_0)^2}\frac{\pi D}{\lambda}$



Results

A0 = 5.961e+01 ± 1.05e+00

X0 =-1.300e+01 ± 5.19e+00

Y0 = 6.964e+01 ± 5.29e+00

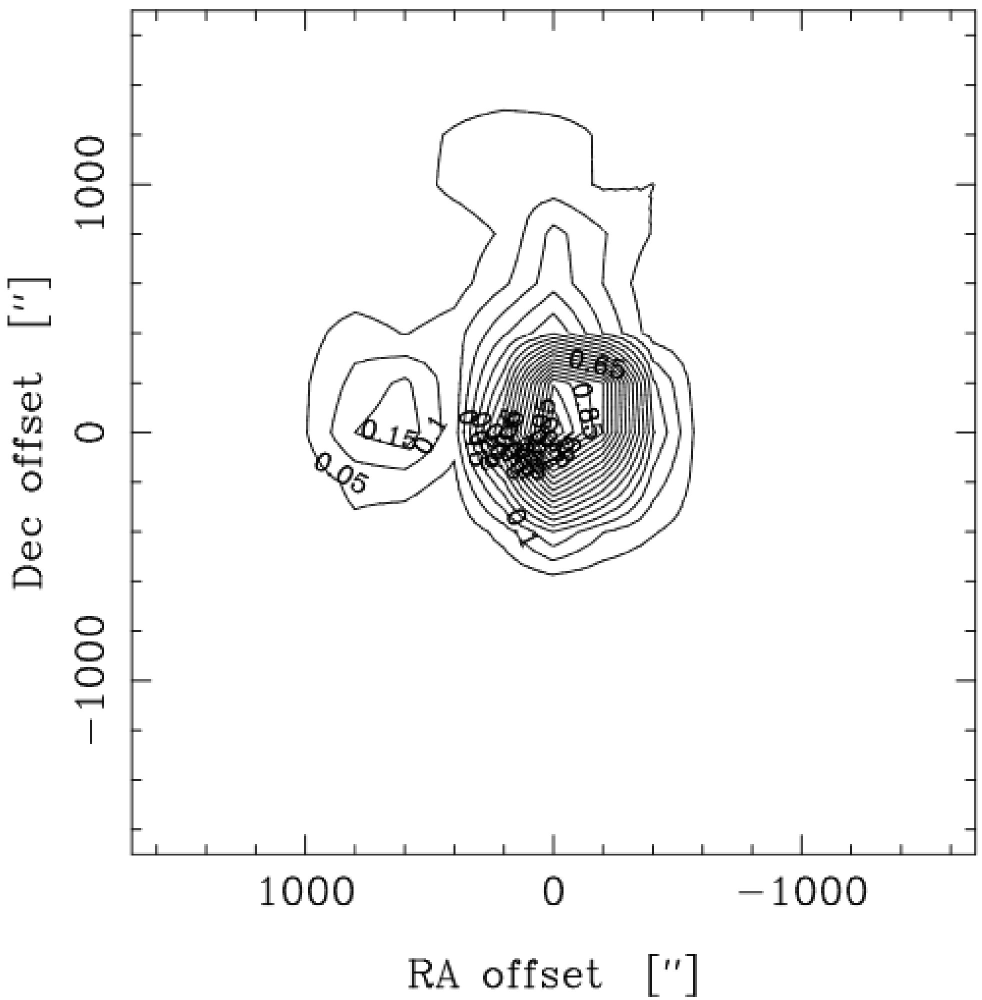
D = 1.943e+01 ± 2.43e-01

FWHM = 712 arcseconds

Observing antenna beams

- Open questions
  - How do beam shapes vary with elevation

    Example : Onsala (6.7 Ghz) at low elevation



Observing antenna beams

- Open questions
  - How do beam shapes vary with elevation
  - Beam offsets between polarizations
  - Dependence on frequency
  - Are polynomial model feasible?
- Only beam information up to the first null is used in the fit. But we need some data points further out to determine bias.
- Integration times should be long enough to average out RFI
- Cover range of elevations but most structure is expected at lower elevations. Eg. 80º, 60º, 40º ,20º, 10º
- Would even lower elevations be possible?
- How much elevation change is expected during the observation of a single map? Small enough to treat it as constant?