Estimation of Uncertainty in Noise Measurements Using Monte Carlo Analysis:

A Practical View

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Low Noise Figure Measurements at Cryogenic and Room Temperatures

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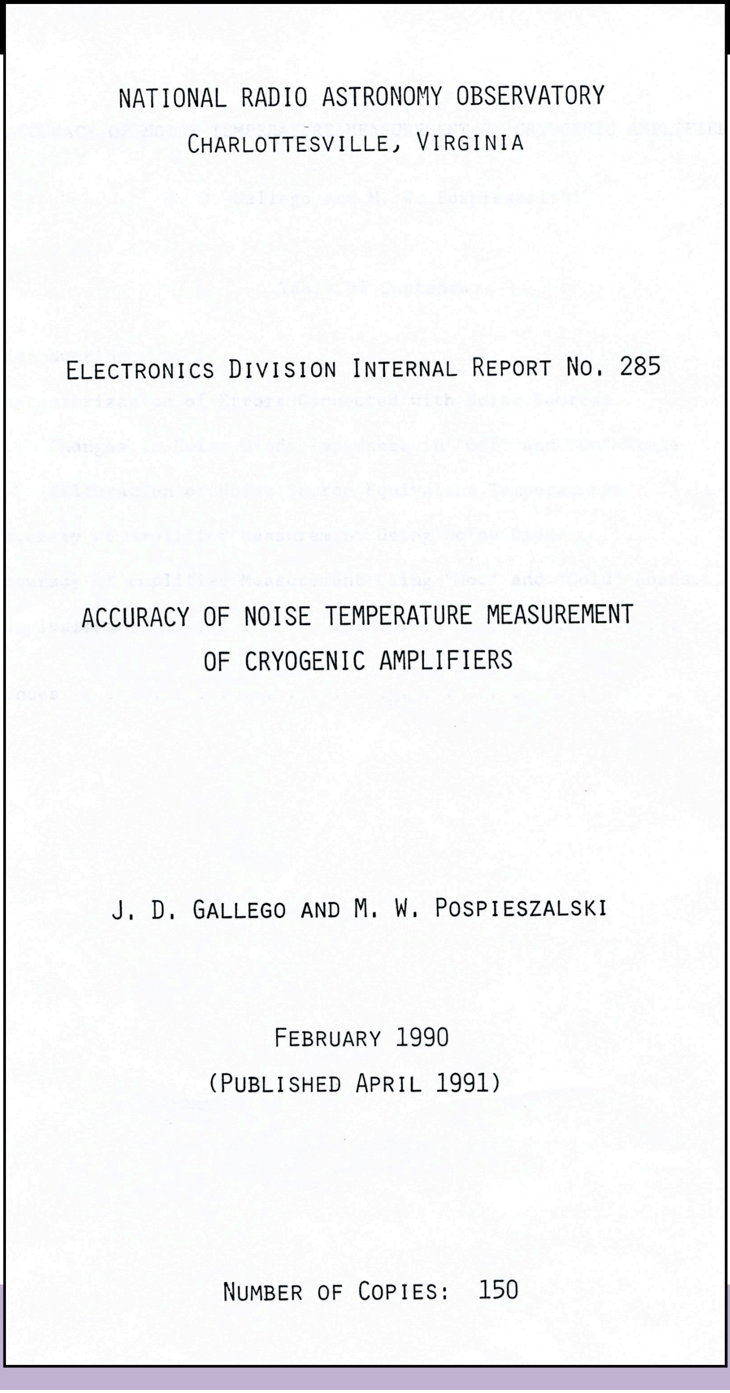

NATIONAL RADIO ASTRONOMY OBSERVATORY
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ACCURACY OF NOISE TEMPERATURE MEASUREMENT
OF CRYOGENIC AMPLIFIERS

J. D. Gallego and M. W. Pospieszalski

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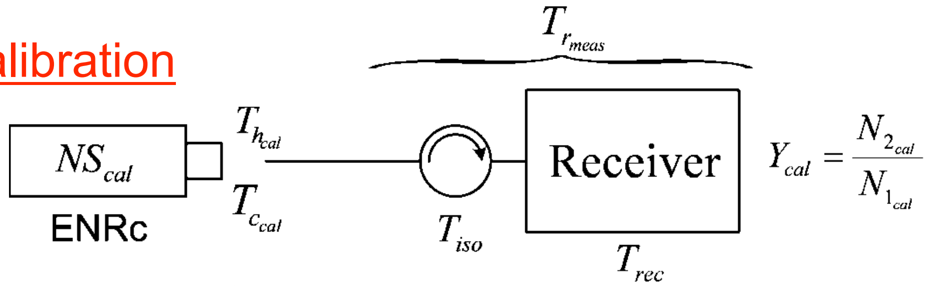
(RadioNet FP7 special edition)

- Outline
  - Introduction
  - Input parameters for Monte Carlo analysis
  - Example 1: Ambient noise, high (15 dB) ENR noise source
  - Example 2: Ambient noise, low (5 dB) ENR noise source
  - Example 3: Ambient noise, hot and cold (LN2) Loads
  - Example 4: Cryogenic noise, cold attenuator method
  - Software
  - Conclusions

Introduction: Monte Carlo method

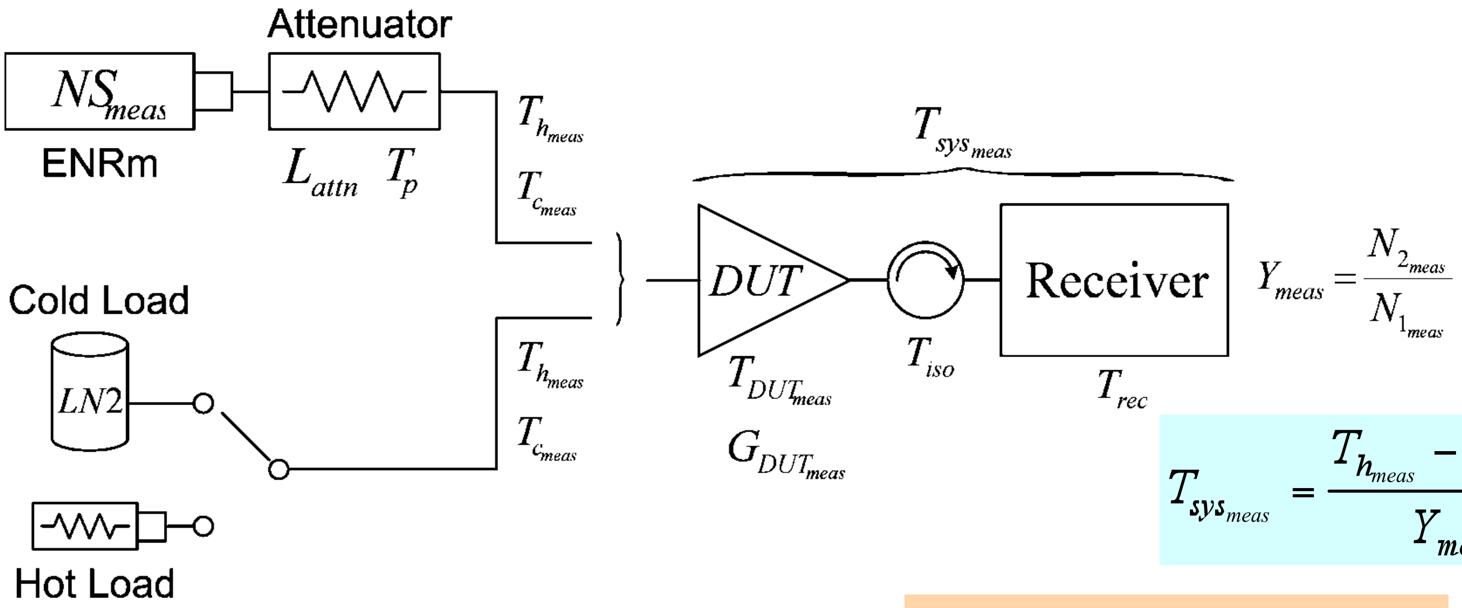
1. Calibration



$$Y_{cal} = \frac{N_{2_{cal}}}{N_{1_{cal}}}$$

$$T_{r_{meas}} = \frac{T_{h_{cal}} - Y_{cal} T_{c_{cal}}}{Y_{cal} - 1}$$

2. Measurement



$$Y_{meas} = \frac{N_{2_{meas}}}{N_{1_{meas}}}$$

$$T_{sys_{meas}} = \frac{T_{h_{meas}} - Y_{meas} T_{c_{meas}}}{Y_{meas} - 1}$$

$$G_{DUT_{meas}} = \frac{N_{2_{meas}} - N_{1_{meas}}}{N_{2_{cal}} - N_{1_{cal}}} \frac{T_{h_{cal}} - T_{c_{cal}}}{T_{h_{meas}} - T_{c_{meas}}}$$

$$T_{DUT_{meas}} = T_{sys_{meas}} - \frac{T_{r_{meas}}}{G_{DUT_{meas}}}$$

Monte Carlo Method

- Statistics over a large number of random trials:
  1. Input values randomly generated (according with the accuracy constraints)
  2. Simulated measurement
  3. Statistics on results → estimation of the accuracy
- Implemented:
  - Accuracy of noise sources and cold temperatures
  - Reflection coefficients
  - Amplifier gain
  - Available gain ≠ Transducer gain
  - Change in Γ (case of noise diodes)
  - Radiometric noise (finite BW and t)
  - Noise parameters (dependence of noise on input Z)
  - Receiver gain accuracy (from calibration to measurement)
  - Receiver non-linearity
- Not implemented (yet):
  - Receiver "drift"
  - Dependence of non-linearity on signal level

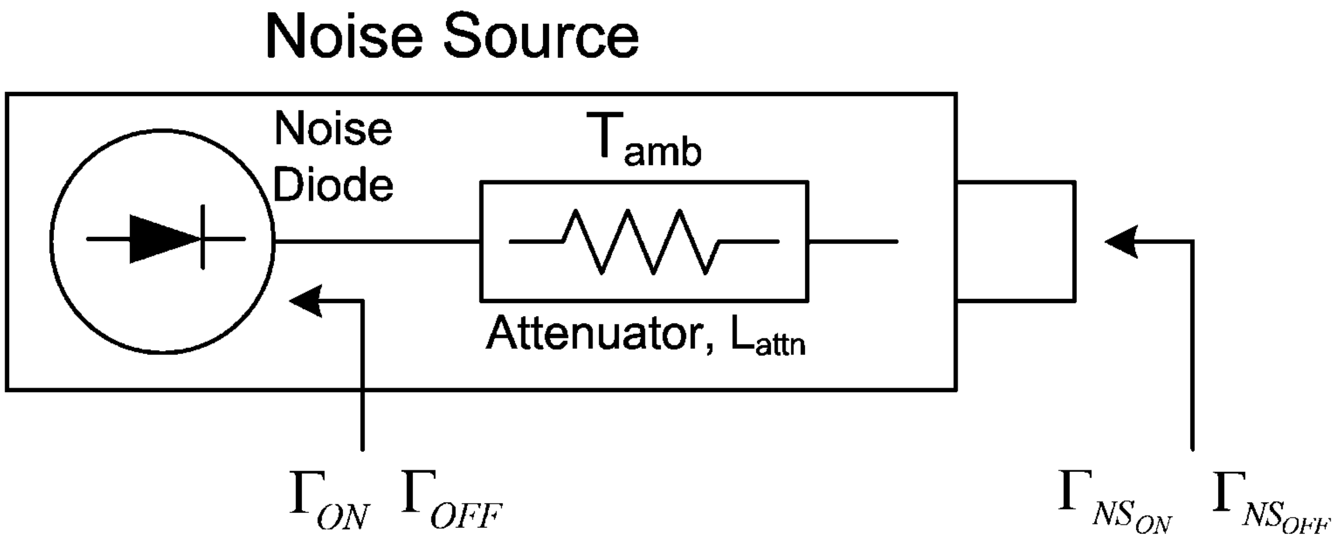
• Input parameters for Monte Carlo method

- A normal distribution is assumed for each parameter affected by inaccuracy (NIST Tech. Note 1297) [4]
  - Mean (μ): nominal value provided by the manufacturer
  - Uncertainty (u_i): provided by the manufacturer (= k·σ)
    - Standard deviation (σ): is the uncertainty divided by the coverage factor
    - Coverage factor (k): provided by the manufacturer or from a guess
    - k = 3 (99.73%), k = 2 (95.45%), k = 1.645 (90%)…
- Reflection coefficients generated as:
  - Magnitude: worst case from measurements or data provided by the manufacturer
  - Phase: randomly generated with a uniform distribution in (0, 2π)
- In solid state NS the states ON and OFF are not independent

$$\Gamma^{r}_{cal_{OFF}} = \Gamma^{r}_{cal_{ON}} + \Gamma_{cal_{diff}} e^{-j\phi^{r}_{cal_{diff}}}$$

- DUT S-parameters generated with random phases
- Receiver noise parameters assuming an isolator at the input
- DUT noise parameters can be included

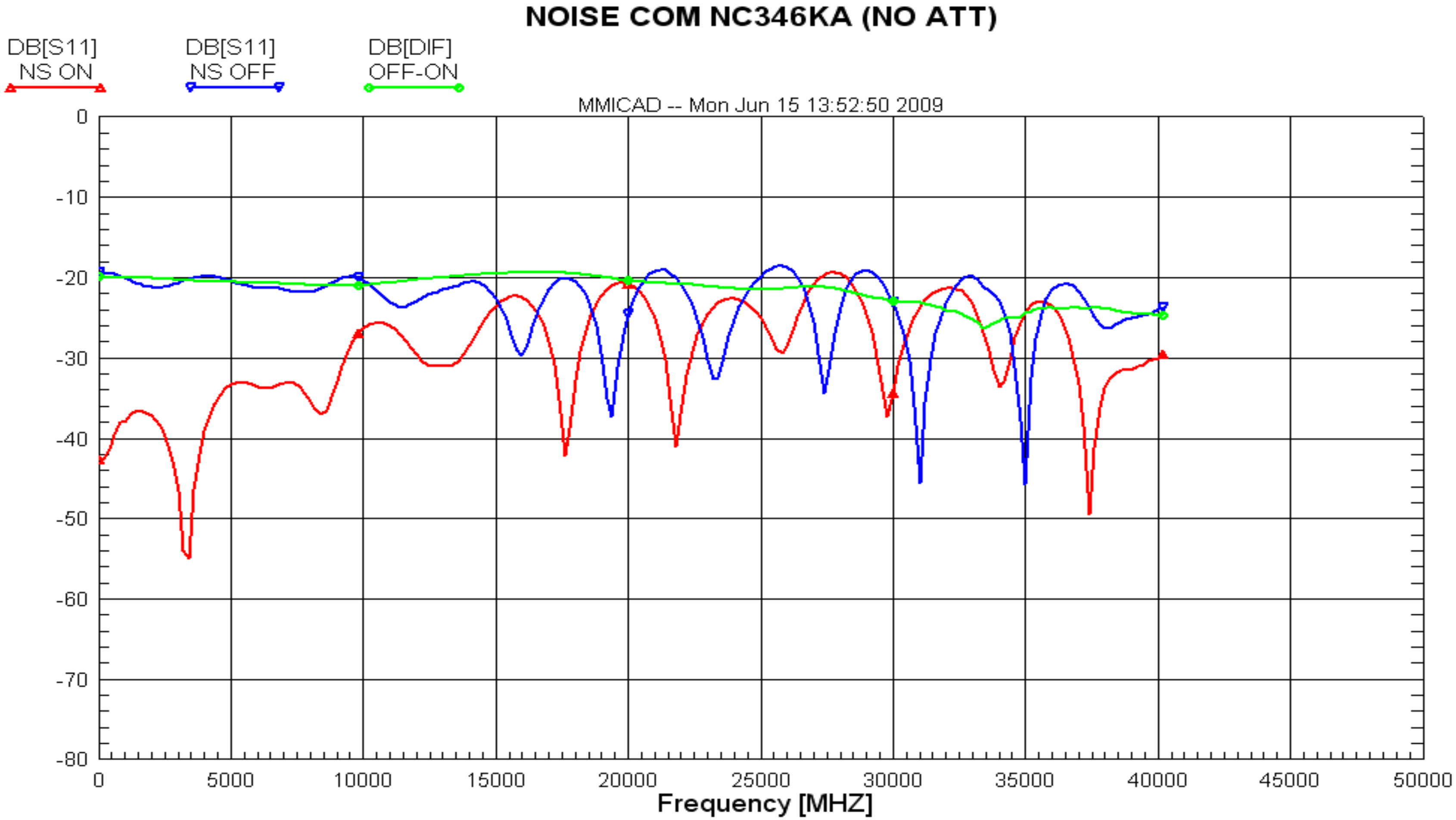
This is how a noise source is built



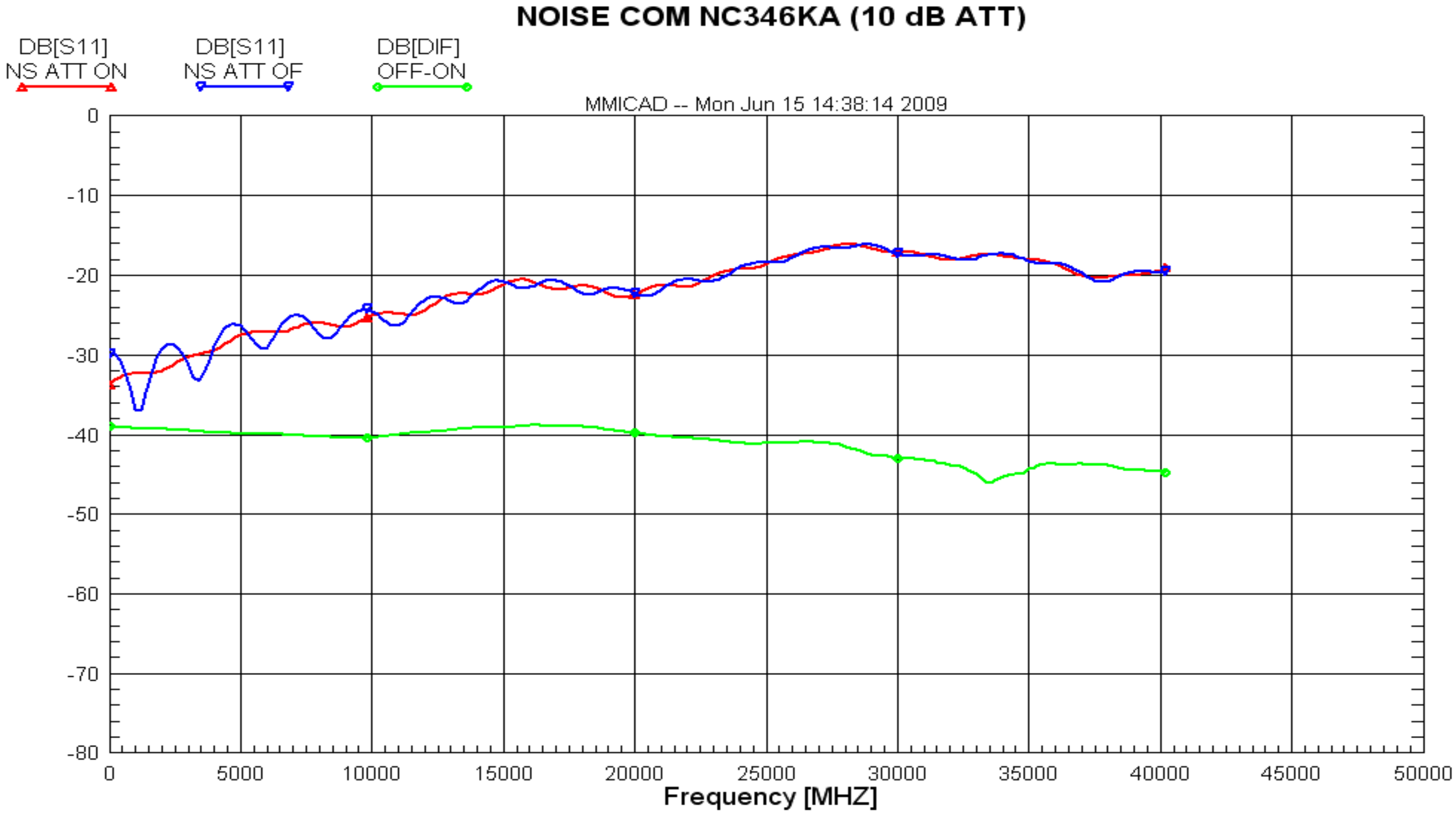
- Large difference in reflection coefficients at diode output $\Gamma_{diff} = \Gamma_{ON} - \Gamma_{OFF}$
- Difference in reflection coefficients reduced by twice L_{attn} at NS output
- $\Gamma_{NSdiff}(dB) = \Gamma_{diff}(dB) - 2 \cdot L_{attn}(dB)$
- The diode noise (when ON) is added to the thermal noise of the attenuattor:
  - $T_{cold} = T_{amb}$ (not T_0)
  - $T_{hot} = T_{amb} + T_0 \cdot 10^{ENR/10}$; (T_0 =290 K by definition)

Example: reflection coefficient of NOISE COM diode noise source

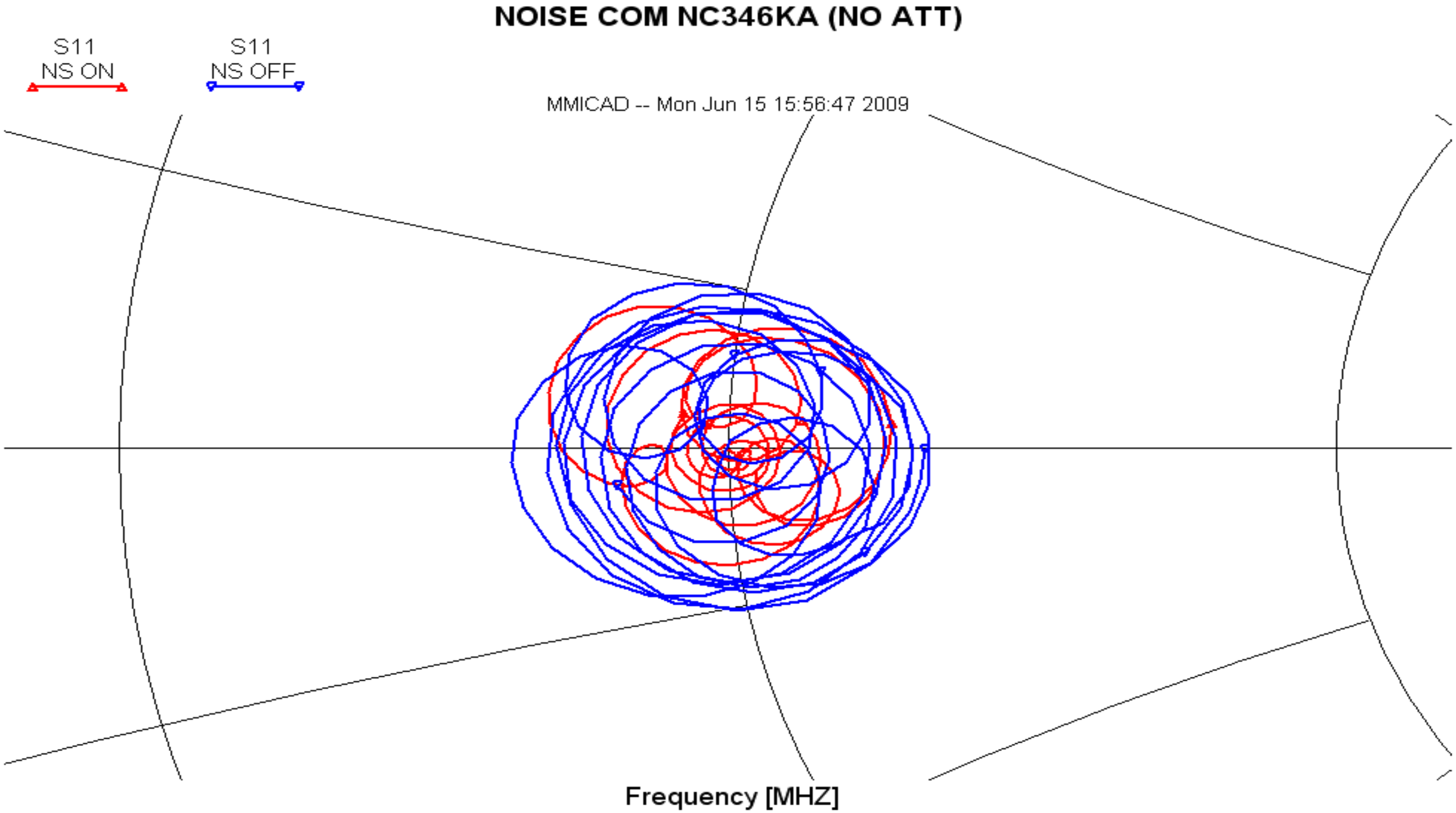
NOISE COM NC346KA (0.1-40 GHz) reflection change



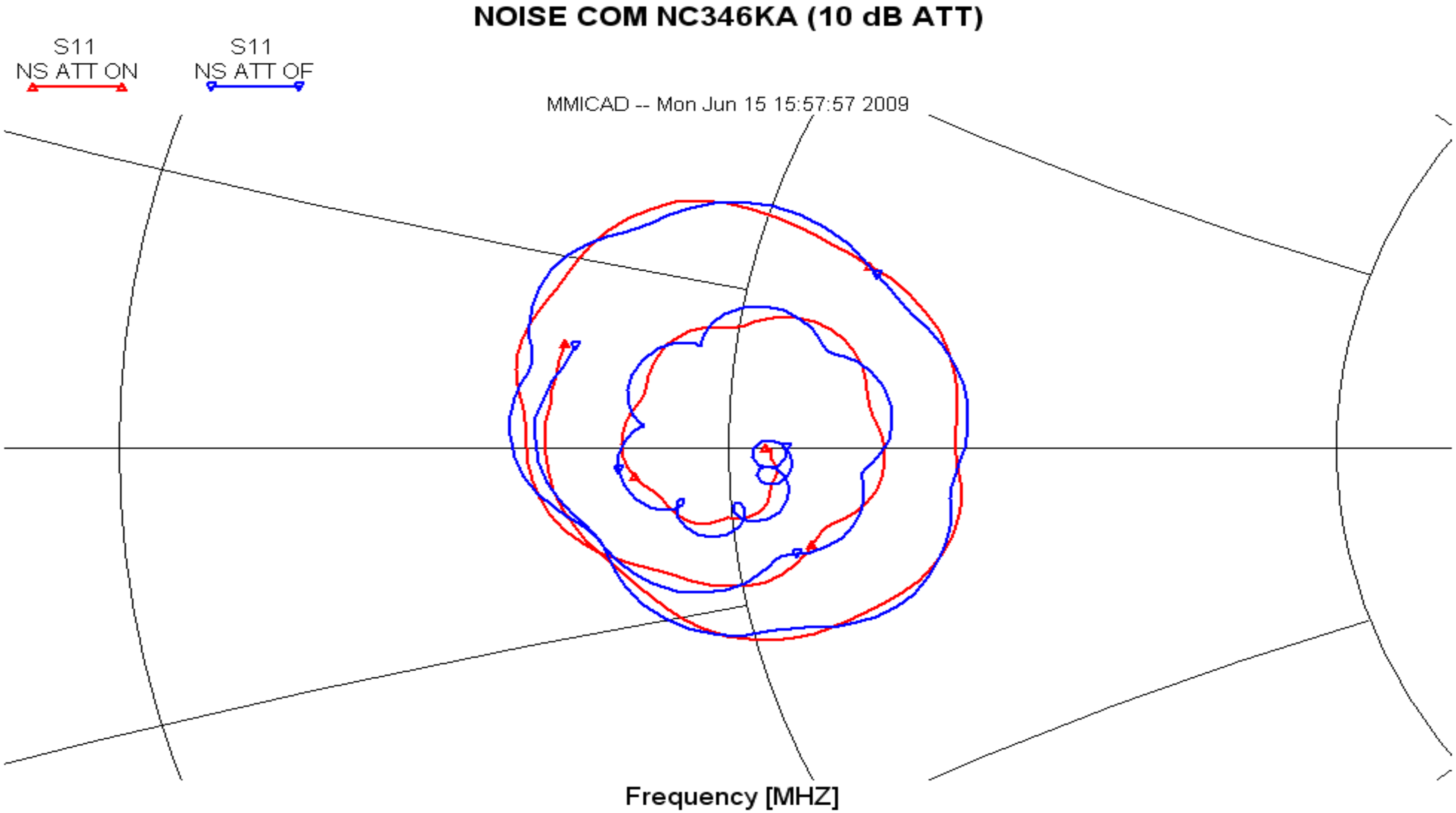
NOISE COM NC346KA + 10 dB ATT. (0.1-40 GHz) reflection change



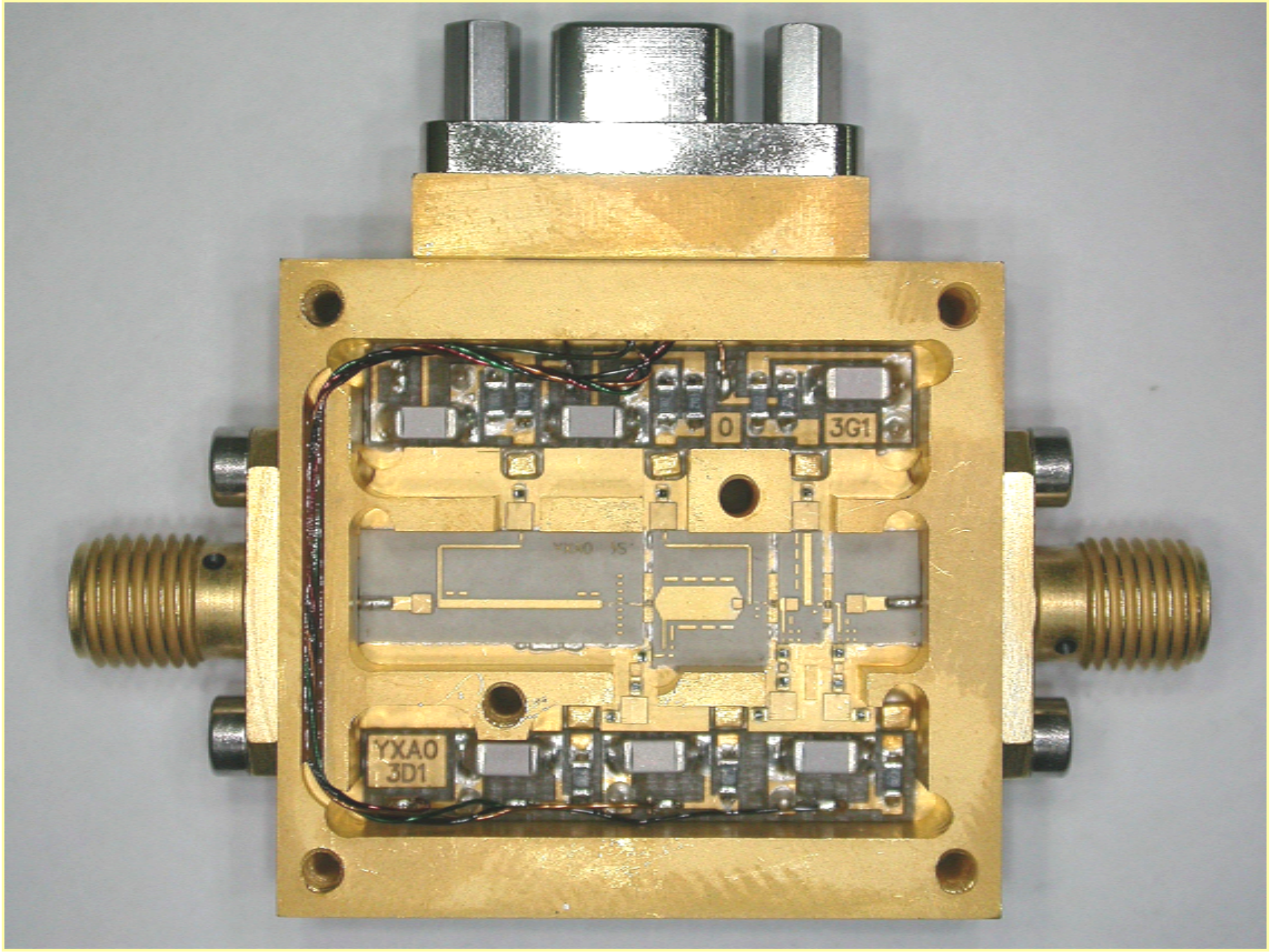
NOISE COM NC346KA (0.1-40 GHz) reflection change



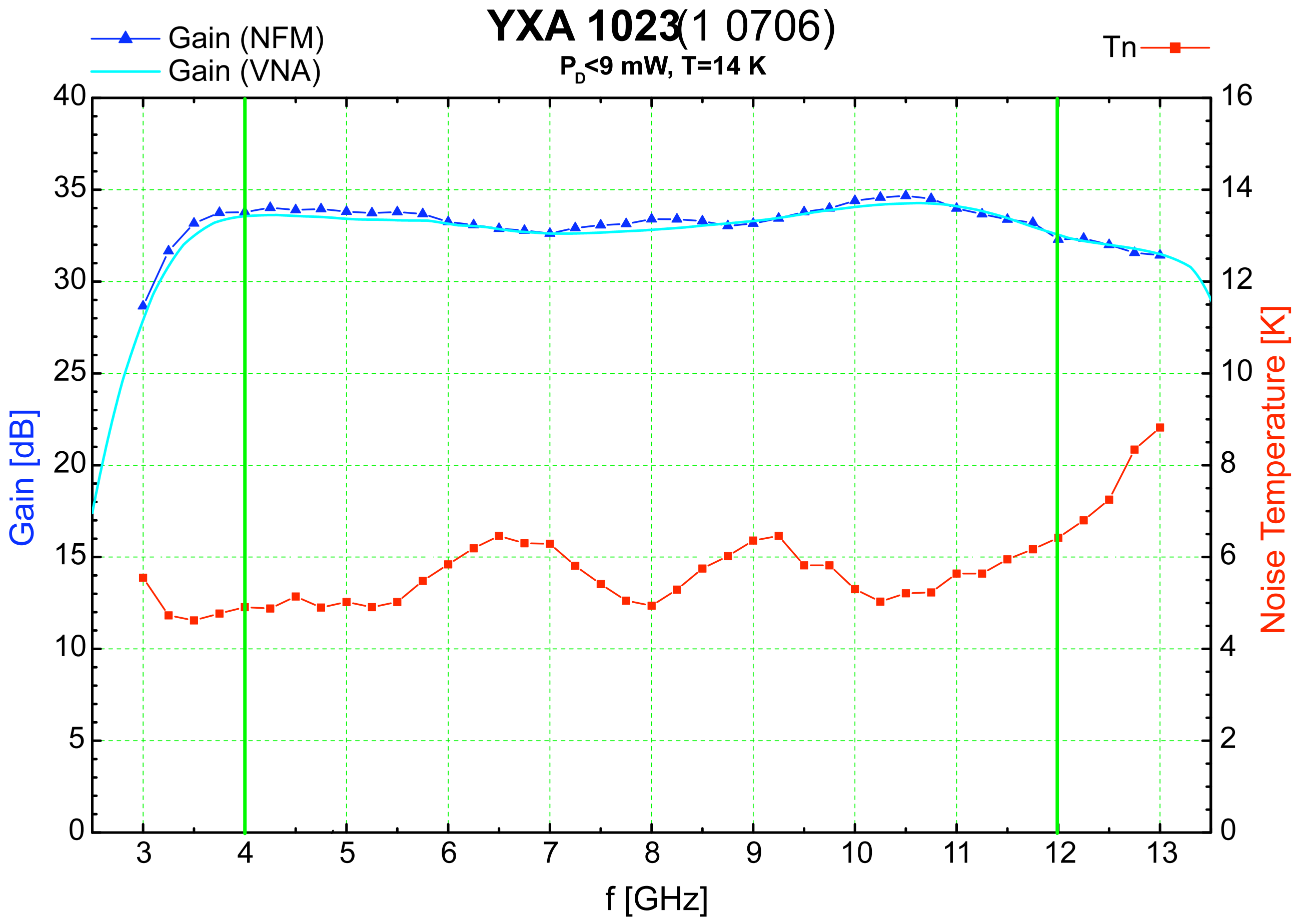
NOISE COM NC346KA + 10 dB ATT. (0.1-40 GHz) reflection change



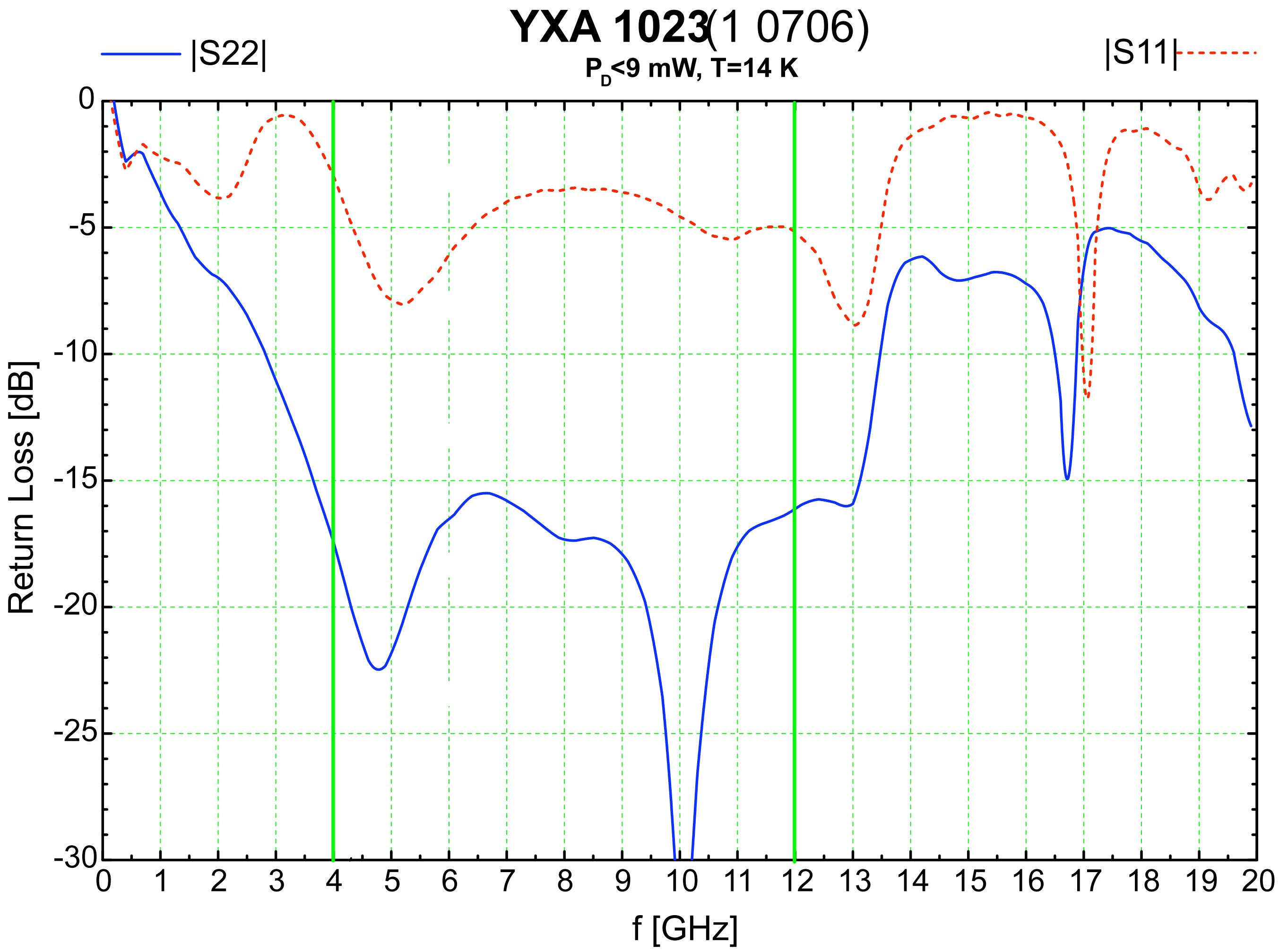
ALMA 4-12 GHz Amplifier
(used for the examples)



ALMA 4-12 GHz Amplifier



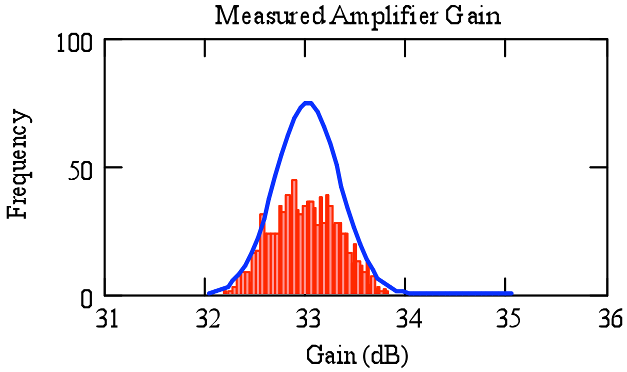
ALMA 4-12 GHz Amplifier

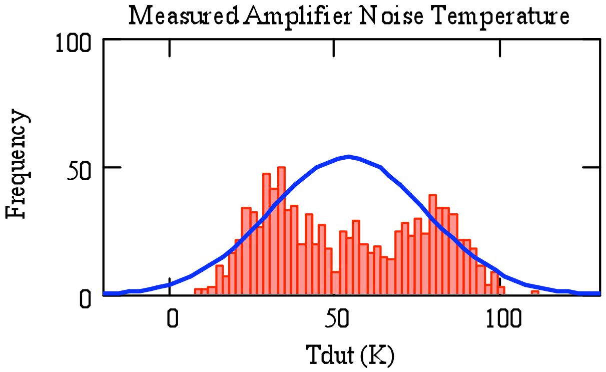


- Example 1: Noise measurement at RT
  - Calibration NS with high ENR (N4002A Agilent Tech., ENR = 14dB)
  - Measurement NS with high ENR (N4002A Agilent Tech., ENR = 14dB)
  - Receiver (N8975A Agilent Tech.) with isolator at input
  - DUT: Amplifier from ALMA Band 9 FI (4 – 12 GHz), results at 8 GHz

Measured Gain of Amplifier (dB)	
Parameter	Value
Mean (μ)	33
Uncertainty (2σ)	0.65

Measured Noise of Amplifier (K)	
Parameter	Value
Mean (μ)	54.65
Uncertainty (2σ)	46.9

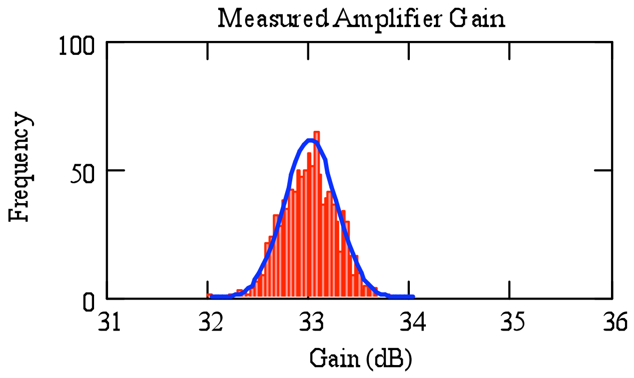


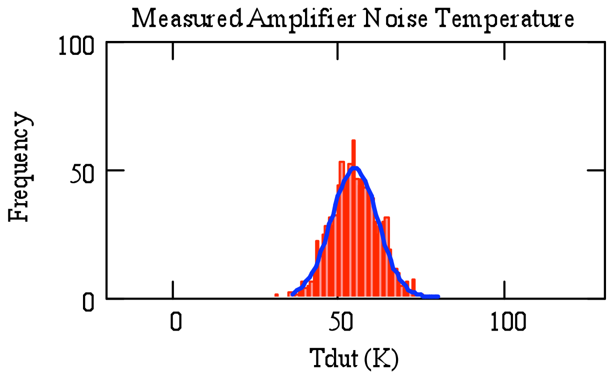


- Example 2: Noise measurement at RT
  - Calibration NS with low ENR (N4000A Agilent Tech., ENR = 5dB)
  - Measurement NS with low ENR (N4000A Agilent Tech., ENR = 5dB)
  - Receiver (N8975A Agilent Tech.) with isolator at input
  - DUT: Amplifier from ALMA Band 9 FI (4 – 12 GHz), results at 8 GHz

Measured Gain of Amplifier (dB)	
Parameter	Value
Mean (μ)	33
Uncertainty (2σ)	0.54

Measured Noise of Amplifier (K)	
Parameter	Value
Mean (μ)	54.2
Uncertainty (2σ)	13.95

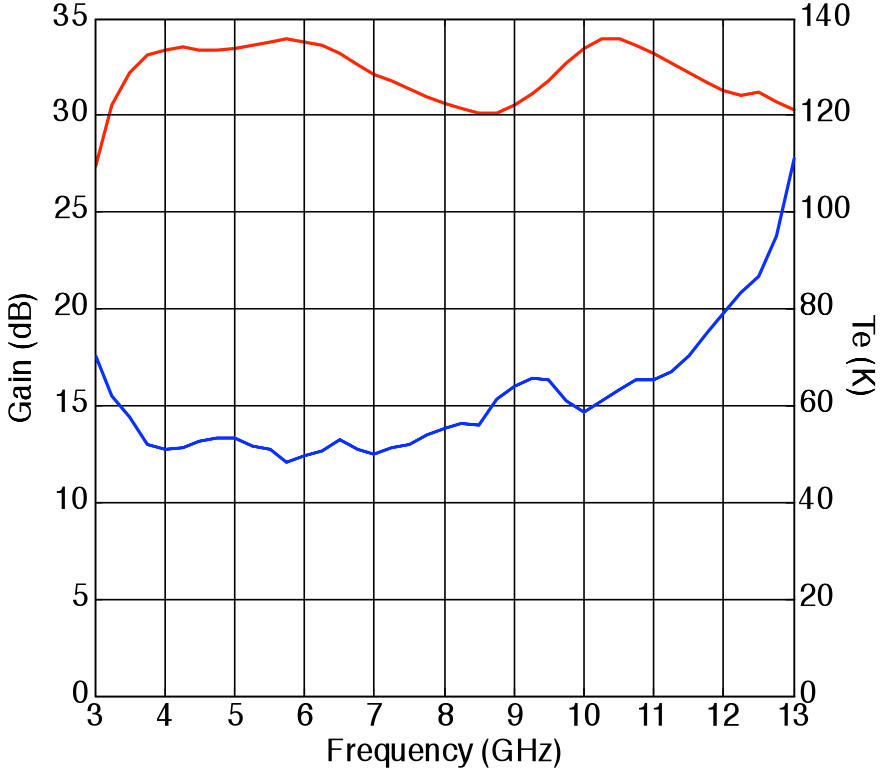




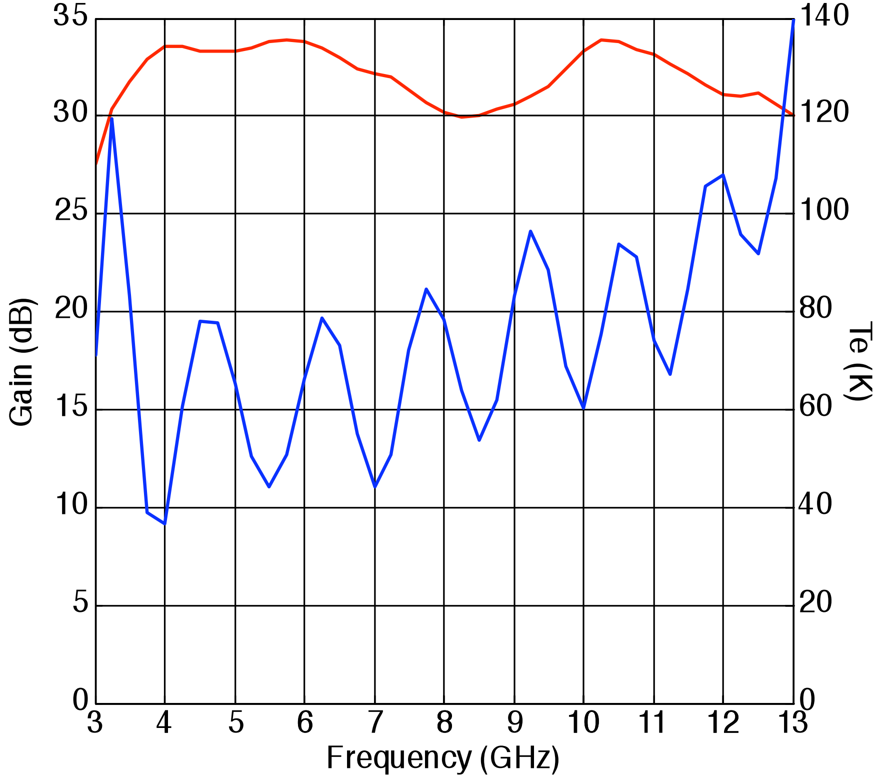
- Comparison of real measurements: Examples 1 and 2

- Comparison of real measurements: Examples 1 and 2

Calibration and measurement with N4000A (low ENR NS)

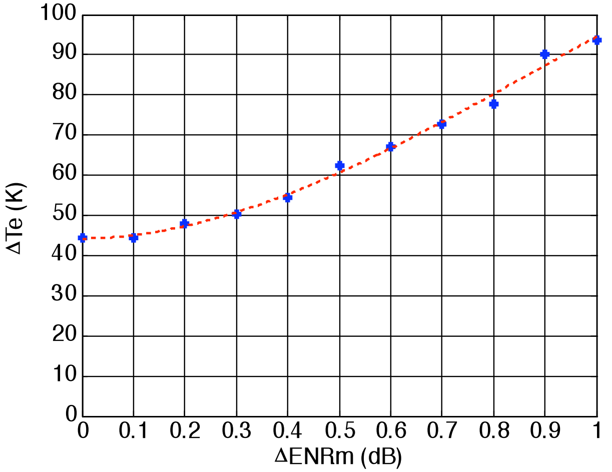


Calibration and measurement with N4002A (high ENR NS)

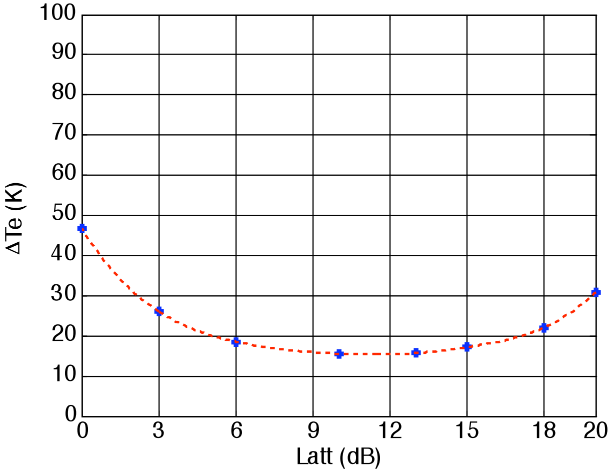


- Some dependences in T_e uncertainty for RT measurements

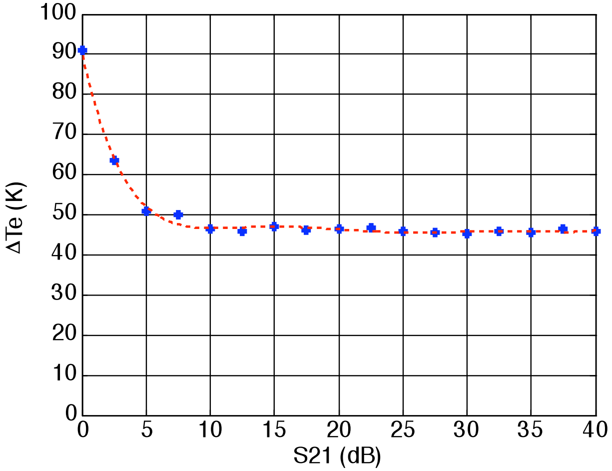
Error in ENR of high ENR meas. NS



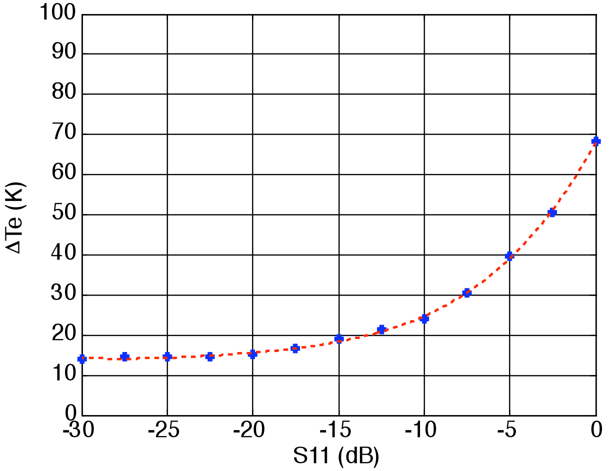
Attenuator after high ENR meas. NS



Amplifier Gain



DUT input reflection coefficient



- Example 3: Measurement at RT with H&C loads
  - Hot and cold loads are independent

$$\Gamma mhmax := 10^{\frac{\Gamma mhmaxdB}{20}} \qquad \phi mh_i := \mathrm{UNIF}(0, 2\cdot\pi) \qquad \Gamma mon_i := \Gamma mhmax\, e^{-j\cdot\phi mh_i} \qquad Tcm_i := \mathrm{NORM}\left(Toc, \frac{\Delta Toc}{kToc}\right)$$

$$\Gamma mcmax := 10^{\frac{\Gamma mcmaxdB}{20}} \qquad \phi mc_i := \mathrm{UNIF}(0, 2\cdot\pi) \qquad \Gamma moff_i := \Gamma mcmax\, e^{-j\cdot\phi mc_i} \qquad Thm_i := \mathrm{NORM}\left(Toh, \frac{\Delta Toh}{kToh}\right)$$

  - Cold load (model MT7118A Maury Microwave)
    - Γ_{mcmax} : SWR = 1.10 (-26 dB) in the 4 – 12 GHz range
    - T_{oc} = 85 K
    - ΔT_{oc} = 1.2 K
  - Hot load (model 2695A Maury Microwave)
    - Γ_{mhmax} : SWR = 1.06 (-30 dB) in the 4 – 12 GHz range
    - T_{oh} = 297 K
    - ΔT_{oh} = 0.5 K

- Example 3: Measurement at RT with H&C loads

Cold load (model MT7118A Maury Microwave)

Hot load (model 2695A Maury Microwave)

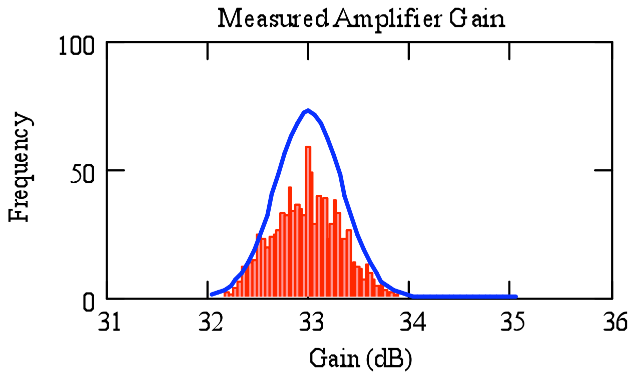
- Example 3: Noise measurement at RT
  - Calibration NS with low ENR (N4000A Agilent Tech., ENR = 5dB)
  - Measurement with Hot and Cold loads (Maury Microwave)
  - Receiver (N8975A Agilent Tech.) with isolator at input
  - DUT: Amplifier from ALMA Band 9 FI (4 – 12 GHz), results at 8 GHz

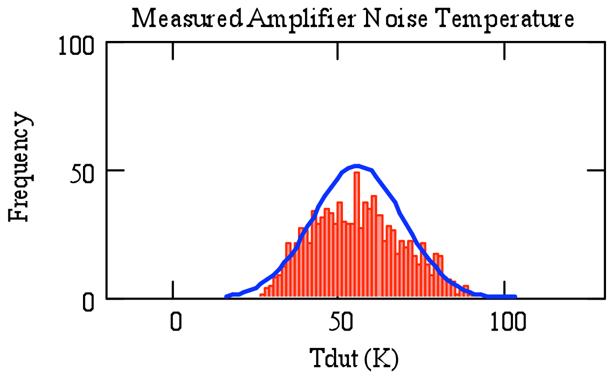
Measured Gain of Amplifier (dB)

Parameter	Value
Mean (μ)	33
Uncertainty (2σ)	0.68

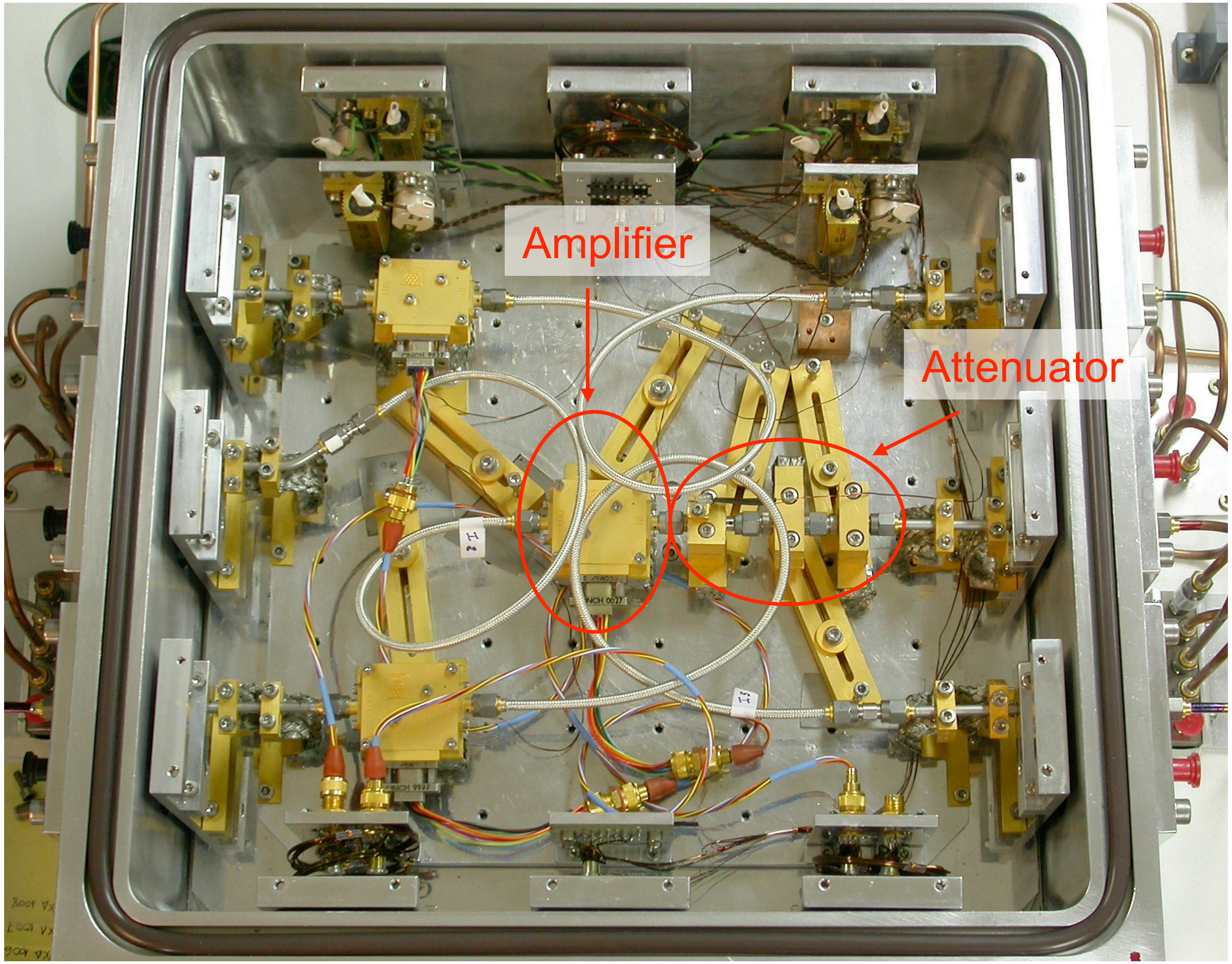
Measured Noise of Amplifier (K)

Parameter	Value
Mean (μ)	55.12
Uncertainty (2σ)	27.56


Measured Amplifier Gain


Measured Amplifier Noise Temperature

- Example 4: Noise measurement at Cryo (T=12.5K)



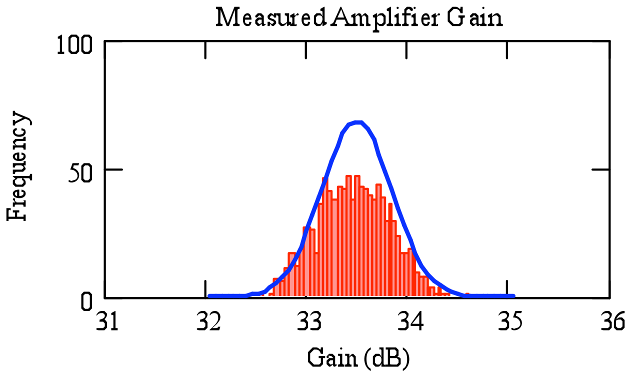
- Example 4: Noise measurement at Cryo (T=12.5K)
  - Calibration NS with low ENR (N4000A Agilent Tech., ENR = 5dB)
  - Measurement NS with high ENR (N4002A Agilent Tech., ENR = 14dB)
  - Attenuation input pad 15 dB (attenuator + connectors +…)
  - Same receiver and DUT as in previous examples, results at 8 GHz

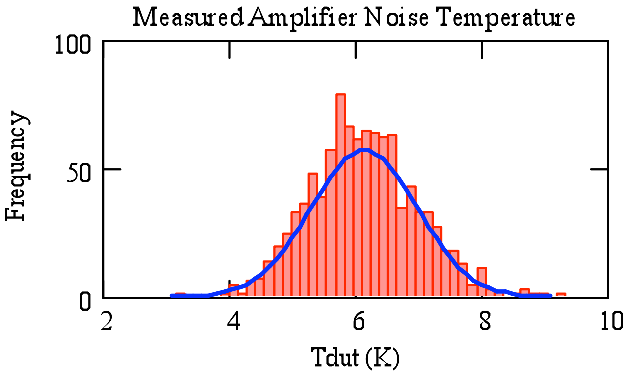
Measured Gain of Amplifier (dB)

Parameter	Value
Mean (μ)	33.5
Uncertainty (2σ)	0.7

Measured Noise of Amplifier (K)

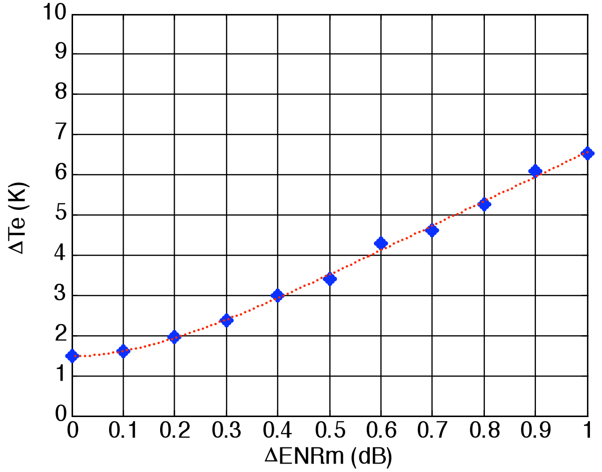
Parameter	Value
Mean (μ)	6.05
Uncertainty (2σ)	1.72



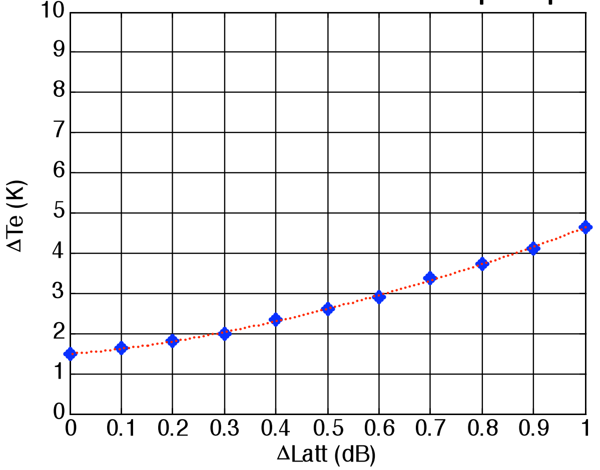


- Some dependences in T_e uncertainty for Cold Attenuator

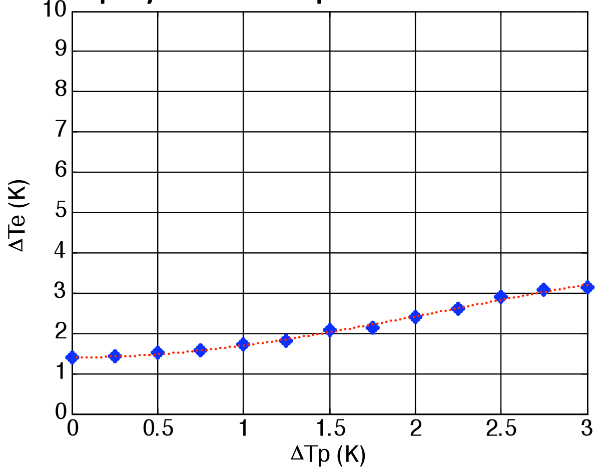
Error in ENR of measurement NS



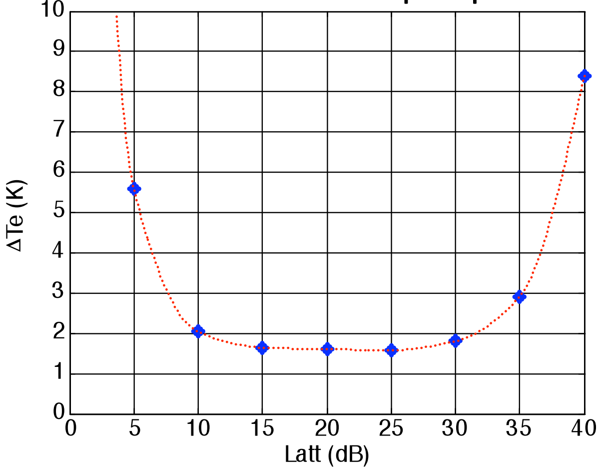
Error in attenuation of input pad



Error in physical temperature of attenuator

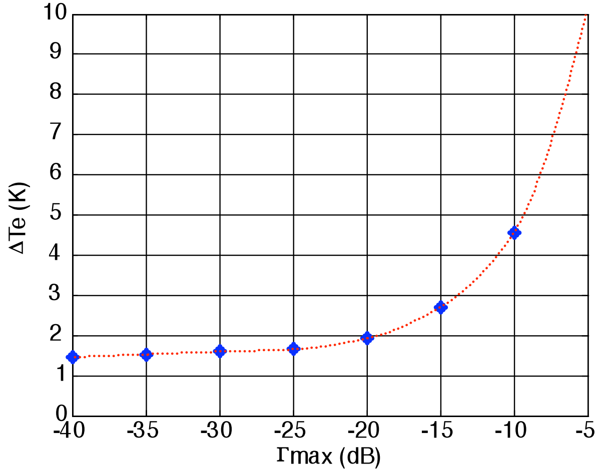


Attenuation of input pad

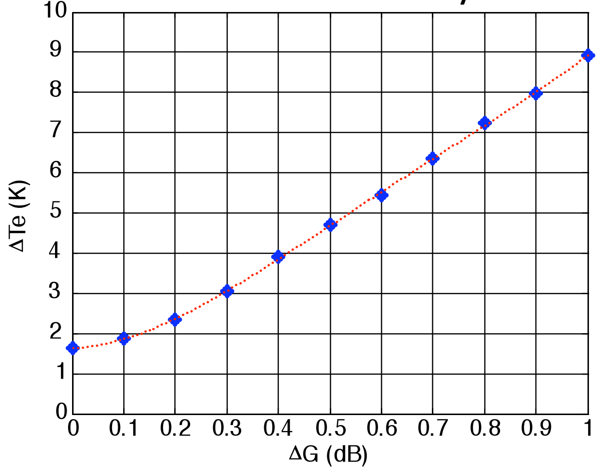


- Some dependences in T_e uncertainty for Cold Attenuator

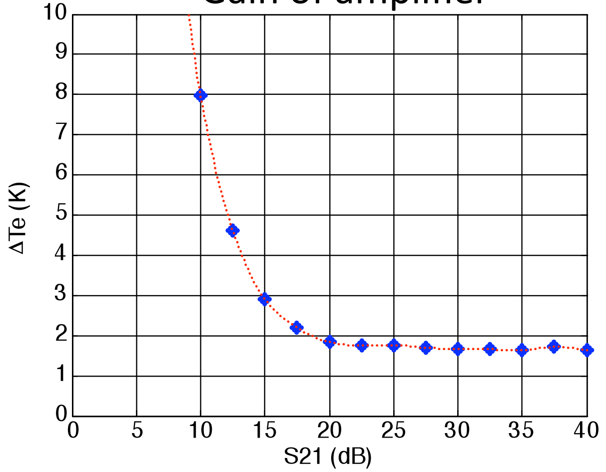
Meas. NS reflection coefficient



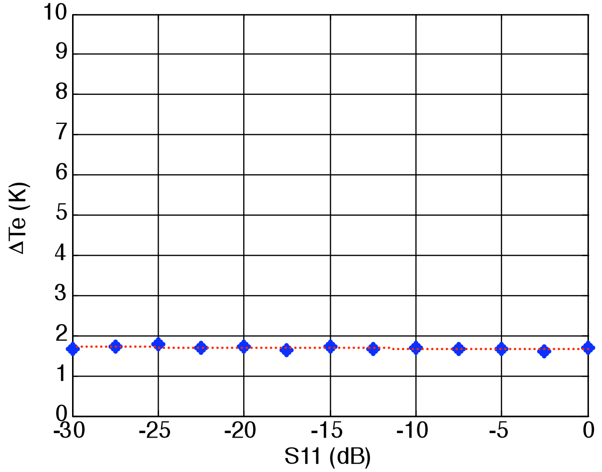
Receiver linearity



Gain of amplifier



DUT input reflection coefficient



- Summary of results

Measured Gain of Amplifier (dB)

Parameter	Room Temperature Tests			Cryo Temp
	Example 1	Example 2	Example 3	Example 4
	High ENR NS	Low ENR NS	H&C Loads	Cold Atten.
Mean (μ)	33	33	33	33.5
Uncertainty (2σ)	0.65	0.54	0.68	0.7

Measured Noise of Amplifier (K)

Parameter	Room Temperature Tests			Cryo Temp
	Example 1	Example 2	Example 3	Example 4
	High ENR NS	Low ENR NS	H&C Loads	Cold Atten.
Mean (μ)	54.65	54.2	55.12	6.05
Uncertainty (2σ)	46.9	13.95	27.56	1.72

- Software
  - Programs developed in ***MathCAD*** and ***Matlab*** to perform calculations
  - Examples 1, 2 and 3 can be directly run with (for examples 1 and 2, set parameters Lattn and ΔLattn to zero, and Tp = Tamb):
    - ***NoiseError_MonteCarlo_ColdAtt.mcd*** (MathCAD)
    - ***NoiseError.m*** (Matlab)
  - Example 4 can be directly run with:
    - ***NoiseError_MonteCarlo_HCLoads.mcd*** (MathCAD)
    - ***NoiseError_HCLoads.m*** (Matlab)
  - The four programs are given to be shared with RadioNET community.

- Software
  - Input data for the programs

NS N4000A

Parameter	Value
ENR (dB)	5.2
ΔENR (dB)	0.14
k_ENR	2
Γmax (dB)	-29
Γdiff (dB)	-48
Tamb (K)	297
ΔTamb (K)	1
k_Tamb	2

Monte Carlo

Parameter	Value
Iterations (n)	1000
k	2

NS N4002A

Parameter	Value
ENR (dB)	14.1
ΔENR (dB)	0.13
k_ENR	2
Γmax (dB)	-24
Γdiff (dB)	-24
Tp (K)	297 (RT) 12.5 (Cryo)
ΔTamb (K)	1
k_Tamb	2
Lattn (dB)	0 (RT) 15 (Cryo)
ΔLattn (dB)	0 (RT) 0.15 (Cryo)
k_Lattn	2

Receiver N8975A

Parameter	Value
Trec (K)	1500
Tiso (K)	297
Γrmax (dB)	-20
B (MHz)	4
t (sec.)	1
ΔGc (dB)	0.17
k_Gc	1.645
ΔG (dB)	0.05
k_G	1.645

Amplifier (DUT)

Parameter	Value
S11 (dB)	-3.5 (RT) -3.5 (Cryo)
S21 (dB)	33 (RT) 33.5 (Cryo)
S12 (dB)	-50 (RT) -47 (Cryo)
S22 (dB)	-14 (RT) -13 (Cryo)
Tmin (K)	45.97 (RT) 3.74 (Cryo)
gn (S)	7.62e-4 (RT) 6.74e-5(Cryo)
Re(Zopt) (Ω)	89.6 (RT) 77.9 (Cryo)
Im(Zopt) (Ω)	15.7 (RT) 71.1 (Cryo)

  - Explanation of all these parameters can be found in the programs

Conclusions

- Practical examples of noise uncertainty calculation with Monte Carlo analysis have been presented both at room and cryogenic temperatures
- Room temperature measurements
  (for high input reflection amplifier)
  - Best: → low ENR noise source
  - Good: → high ENR noise source + attenuator or isolator
  - Not as good: → hot and cold lab standards (uncertainty very dependent on Γmax)
  - Bad: → high ENR noise source
- Cryogenic temperature measurements
  - Cold attenuator method is good for high input reflection amplifiers
  - High DUT gain needed to minimize contribution of (high) receiver noise
  - ENR and cold temperature critical for accuracy (0.2 dB per K and 1 K per K respectively)
  - Effect of noise parameters contributes ~0.2 K to the total uncertainty 2σ (1.72 K)
  - DUT's S11 does affect S21 uncertainty but does not affect Te uncertainty

- References

  - [1] “Fundamentals of RF and Microwave Noise Figure Measurements”, Application Note 57-1. Agilent Technologies. 2006
  - [2] “Noise Figure Measurement Accuracy – The Y-Factor Method”, Application Note 57-2. Agilent Technologies. 2004
  - [3] J. D. Gallego and M. W. Pospieszalski, “Accuracy of Noise Temperature Measurement of Cryogenic Amplifiers”, Electronics Division Internal Report No. 285, NRAO, Charlottesville, VA. 1991
  - [4] B. N. Taylor and C. E. Kuyatt, “Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results”, NIST Technical Note 1297. 1994 Edition.