Proposed phase equation to be used in fringe fitting.

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The generic expression for visibilities for a single baseline as a function of frequency ν and time t is

$$V(\nu, t) = A(\nu, t) \exp(i\phi[\nu, t]),$$

where A and ϕ are amplitude and phase respectively. If we assume that the contribution due to source structure has been divided out, A will be equal to unity. The phase expression in this case is all that is of interest. It is

$$\phi(\nu, t) = \phi(\nu_0, t_0) + 2\pi\nu\ \tau(t).$$

Here ν_0 and t_0 are reference values of these quantities, and τ is the delay offset between signals from the antennas of that baseline at the point at which they are correlated. Note that all frequency-dependent contributions, for example from the ionosphere, have been neglected for the moment. It is also assumed that terms quasi-proportional to time, such as differences between station frequency references, have already been calibrated out.

Since τ is a continuous function of t it is desirable to parameterize it in some way, probably as a weighted sum of basis functions $T_i(t)$:

$$\tau(t) \sim \sum_i^N \theta_i T_i(t, t0).$$

An example of this is the Taylor series

$$\tau(t) \sim \tau(t0) + (t - t0)\ \tau'(t0) + 0.5(t - t0)^2\ \tau''(t0) +$$

etc.

The method 'solveDelay' proposed for fringe fitting in CASA will produce estimates of the coefficients θ_i for some N. It should also output $\phi(\nu_0, t_0)$, ν_0 and t_0.

It is planned that several flavours of solver may follow one another in sequence. In such a chain of solvers, the delay solution obtained by solver $j-1$ is applied to the data sent to solver j. Solver j will then in effect estimate the difference between the true delay function and the τ estimated from solver $j-1$. Thus the values of θ_i together with $\phi(\nu_0, t_0)$, ν_0 and t_0 obtained by solver $j-1$ should also be sent as input parameters to solver j, so j can add this to its own solution before outputting it.

One could conceive of a delay object which returned either a parameterization or a sequence of delay samples, but for now it is arguably better to 'hard-wire' a simple parameterization, such as Taylor expansion to order 1. In this case each solver will be supplied with t_{j-1}, ϕ_{j-1}, τ_{j-1} and τ'_{j-1} (each with a default value of zero) and solve for ϕ_j, τ_j and τ'_j in the expression

$$\phi(\nu, t) = \phi_j - \phi_{j-1} + 2\pi\nu(\tau_j - \tau_{j-1} - [t_j - t_{j-1}]\tau'_{j-1} + [t - t_j][\tau'_j - \tau'_{j-1}]).$$