Fundamentals of Noisy Networks

Sander Weinreb, California Institute of Technology

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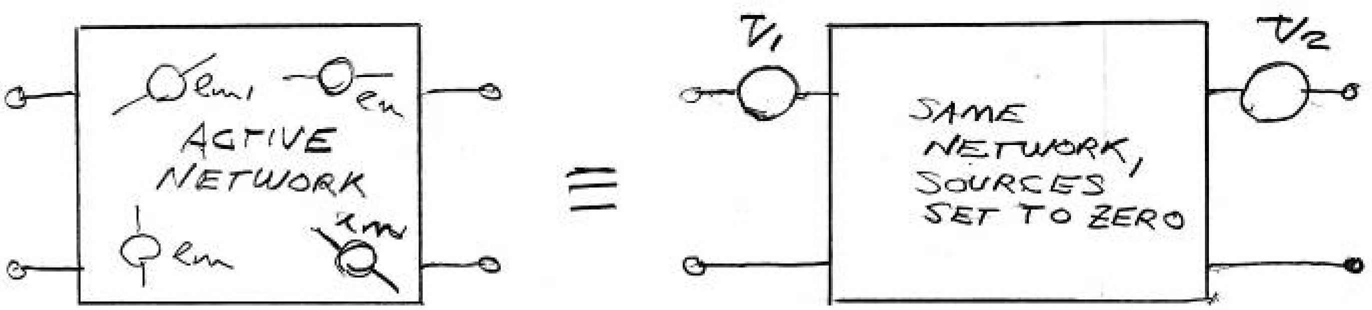
Topics

1. Choose Your Weapons (basis and choice of noise parameters)
2. It Takes Two to Tango (correlation depends on choice of variables)
3. Take your Pick (noise and gain match)
4. Obey the Laws
    a) The essence of N (not Rn)
    b) The essence of Tcasmin (feedback)
5. Noise measurements
6. HEMT and SiGe LNA Examples
7. System noise
8. Noise questions and answers
9. References

Basis of Noisy Network Theory

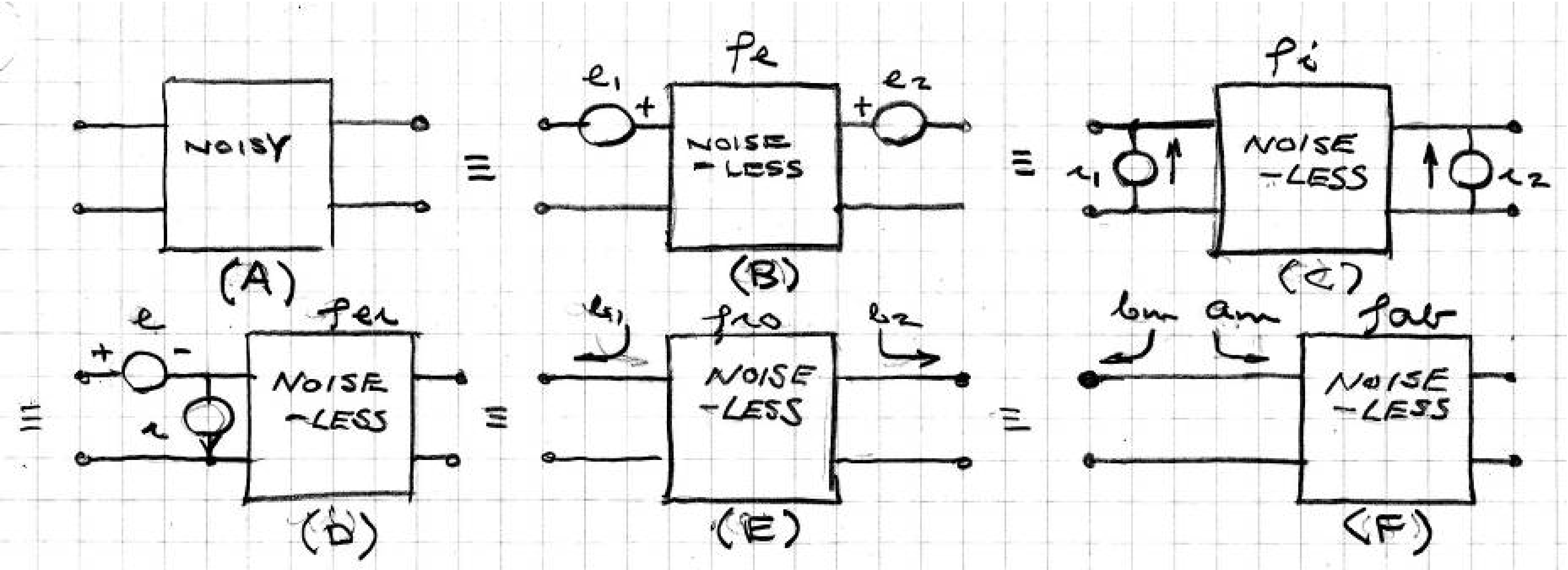
- The basis is the Thevenin theorem (Helmholtz, 1853, Thevenin, 1883, see Wikipedia, 2009) which states that any two-port network containing sources can be represented by the network with sources (series voltage or shunt current) added at the terminals.
- This theorem is independent of the waveforms or number of the internal sources; they can always be represented by two external sources.
- Thus all noise of any source (thermal, shot, avalanche, etc) in a network can be represented by two terminal noise sources. For noise sources the complex correlation coefficient, ρ, between the two sources must be specified, leading to 4 real numbers to describe all noise in the network.

ρ ≡ correlation coefficient



Choose Your Weapons!

- There are many sets of 4 parameters to specify a noisy network and the choice depends upon what is known about the internal sources and upon the application
- For FET or HEMT noise analysis, (C) is most often applied,. for op amps (D) is given in data sheets, for arrays systems studies, (E) is most meaningful
- Once one set is known, transformation to another set is straight-forward



Understanding Waves

- At any reference plane voltage and current are related to forward wave complex amplitude, a, and backward wave b, by:

$$V = (a+b) * \sqrt{Zn} \qquad I = (a-b) / \sqrt{Zn}$$

where Zn is the normalization impedance

- Zn is usually equal to the interconnection transmission line characteristic impedance, Zo, often 50 ohms, or for differential connections 200 ohms. When Zn = Zo, a and b only change phase if the reference plane is changed.
- Wave noise parameters (i.e. (E) and (F) in previous figure) change when Zn is changed but Tn does not change.
- An important case for (E) it that the input-to-output noise wave correlation coefficient, $\rho_{io} = 0$ when an amplifier has been designed for both noise and gain matched to Zn. [see Wedge, et al, 1992]. To be discussed further in a subsequent slide.

It Takes Two to Tango!

- Note that correlation depends upon which two sources we are referring to. Thus the correlation coefficients between input and output currents in the previous figure does not imply correlation between input and output voltages or waves.

$$\rho_i \neq \rho_v \neq \rho_{io}$$

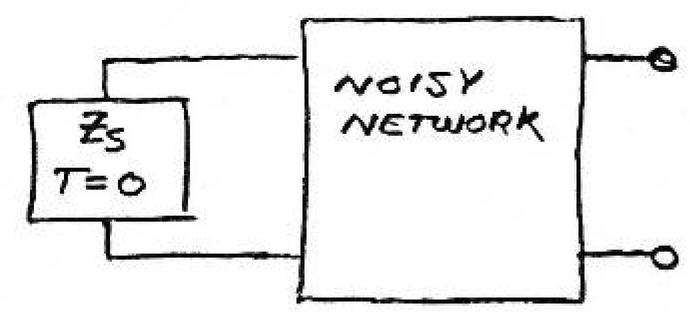
- The term "correlated noise" is often confused. We must specify which two variables we are referring to.

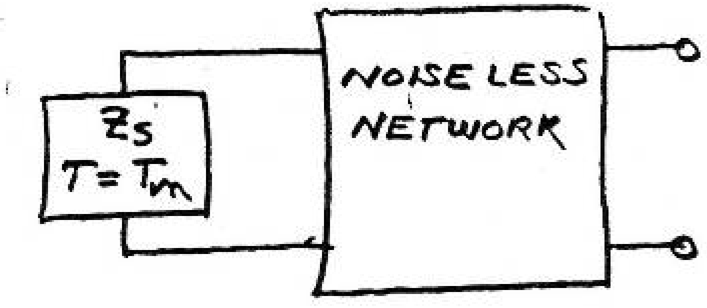
Most Common Microwave Noise Representation

- The noise of microwave amplifier is usually specified by the noise temperature, Tn, that must be added to the source generator to represent the noise in the amplifier. The noise figure is given by

$$NF = 10*\log(1+Tn/290)$$

- Tn is a function of the source impedance Zs and there is a noise parameter, Zopt, which minimizes Tn to give a key 3rd parameter, Tmin. A fourth number is needed to complete the 4 parameter set and this is usually Rn or N to specify the increase in noise if Zs ≠ Zopt.





Noise Temperature, Tn, for any Source Impedance, Zs

- Tn as a function of 4 noise parameters (Tmin, Zopt, and N) and either the source impedance, Zs, source admittance, Ys, or reflection coefficient, Γs is given below.
- The “criticalness” parameter is given by, N = Rn*Gopt, where Rn is more commonly specified, and Gopt is the optimum source conductance, *Re* (Yopt).

$$T_n = T_{min} + N T_0 \frac{|Z_s - Z_{op}|^2}{R_s R_{op}}$$

$$T_n = T_{min} + N T_0 \frac{|Y_s - Y_{op}|^2}{G_s G_{op}}, \quad Y_s = Z_s^{-1} \quad Y_{op} = Z_{op}^{-1}$$

$$T_n = T_{min} + \frac{4 N T_0 |\Gamma_s - \Gamma_{op}|^2}{(1 - |\Gamma_s|^2)(1 - |\Gamma_{op}|^2)}, \quad \Gamma_{op} = \frac{Z_{op} - Z_0}{Z_{op} + Z_0}$$

The Essence of N

- The dimensionless noise parameter, N, has some interesting properties
- N, as well as Tmin, is invariant to lossless input networks. The N of a packaged transistor is the same as the N of a chip. It is not changed by a length of transmission line. [Lange, 1967]
- N is not changed by putting transistors in parallel or changing the width of the transistor
- N is known to within a factor of 2 if Tmin is known (To = 290K).

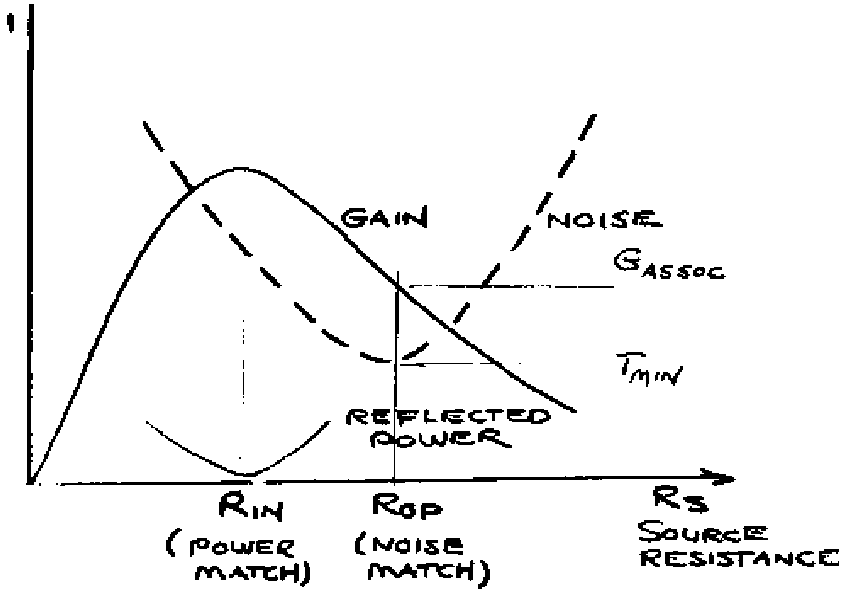
$$\frac{T\min}{4 \bullet To} \leq N \leq \frac{T\min}{2 \bullet To}$$

[See Pospieszalski for proof.]

- For the SiGe bipolar transistors investigated by Bardin, N is close to the upper limit in the above equation
- Noise parameters (usually using Rn) are sometimes published which violate the lower limit above due to errors in the measurements

Noise Match and Gain Match Are Not for Same Zs

- Zopt ≠ Zin* which increases the sensitivity of either Tn or gain to antenna impedance.
- Remedies are: 1) isolator, 2) balanced amplifier, or 3) feedback



- An important case for phased-arrays is that the input-to-output noise wave correlation coefficient, ρio = 0 when an amplifier has been designed for both noise and gain matched to the wave normalization impedance, Zn.

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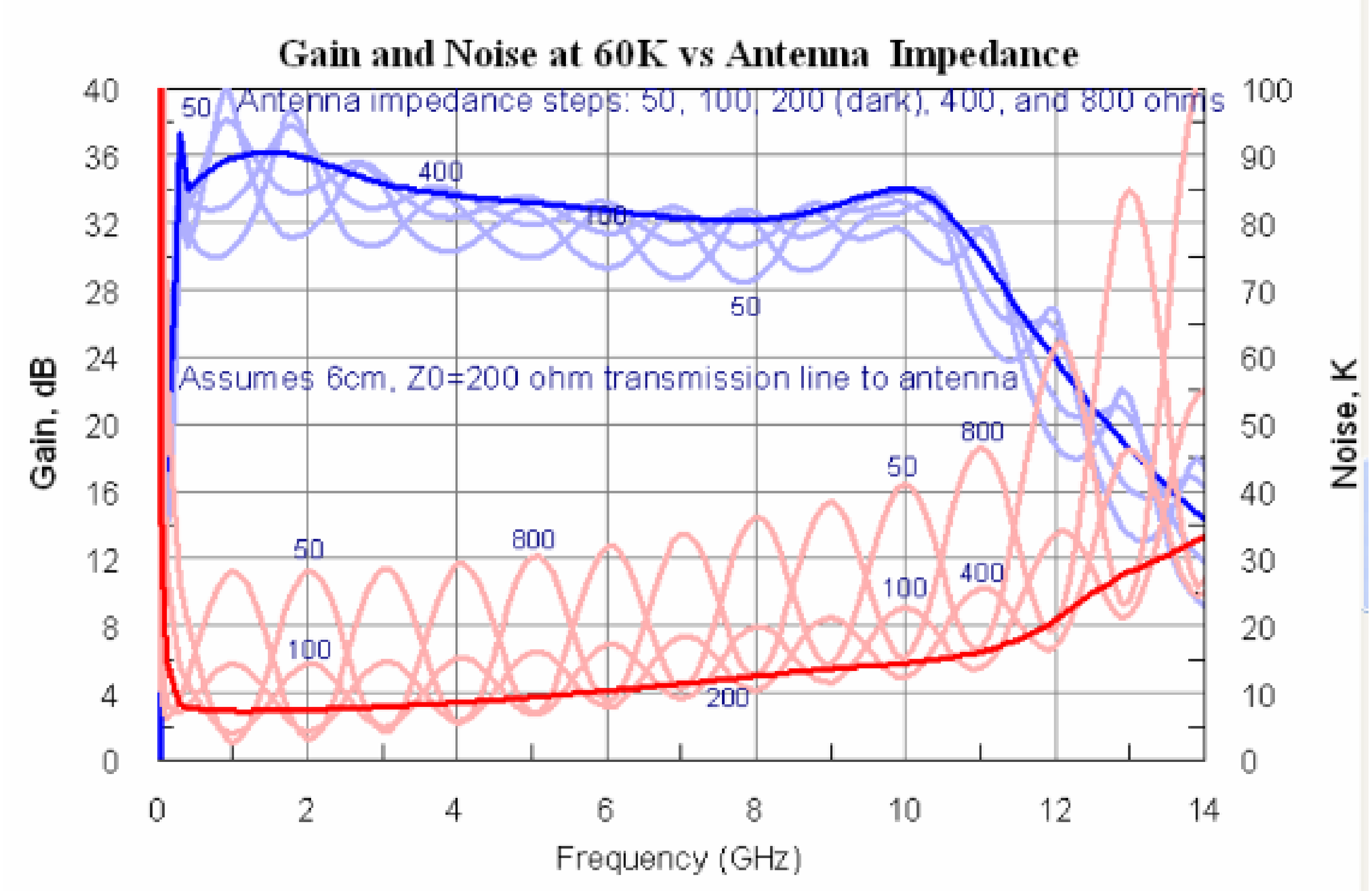
Effect of Mismatched Antenna on Noise and Gain of an LNA

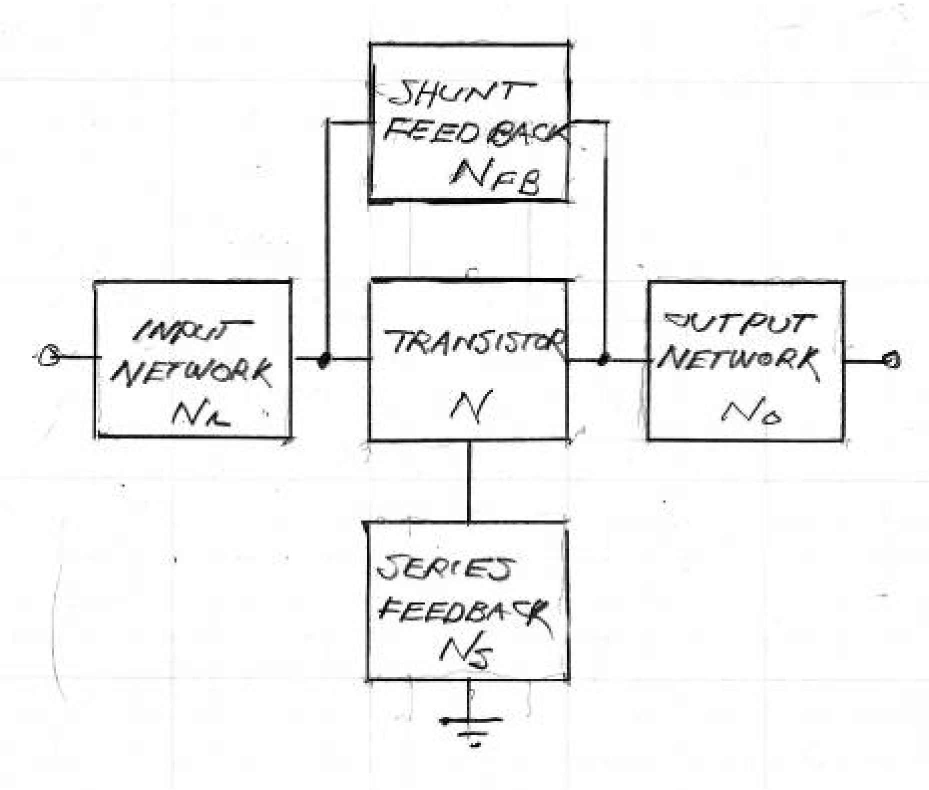
- Modeled gain and noise of a differential InP LNA at 60K as a function of antenna impedance driving a 6cm long transmission line with Zo = 200 ohms
- At 2 GHz VSWR of 2:1 causes noise to vary 4K to 14K and gain to vary +/- 2dB
- At some frequencies noise is improved because Zopt is not equal 200 ohms.



Obey the Laws!

What limits the overall network performance in terms of an embedded network (i.e. a transistor) performance?

Very powerful theorems regarding the effect of feedback on the noise and gain of amplifiers were published by H. Haus in the 1958-1960 era. The papers are not easily understood and are often ignored in the current literature.



Noise Measure and Tcasmin

- Tmin is not the best figure of merit of a low noise transistor since it does not take gain into account.

- Feedback can reduce Tmin to zero! Proof: Consider a feedback network that directly connects input to output and does not connect to the transistor. Tmin =0 but Gain =1

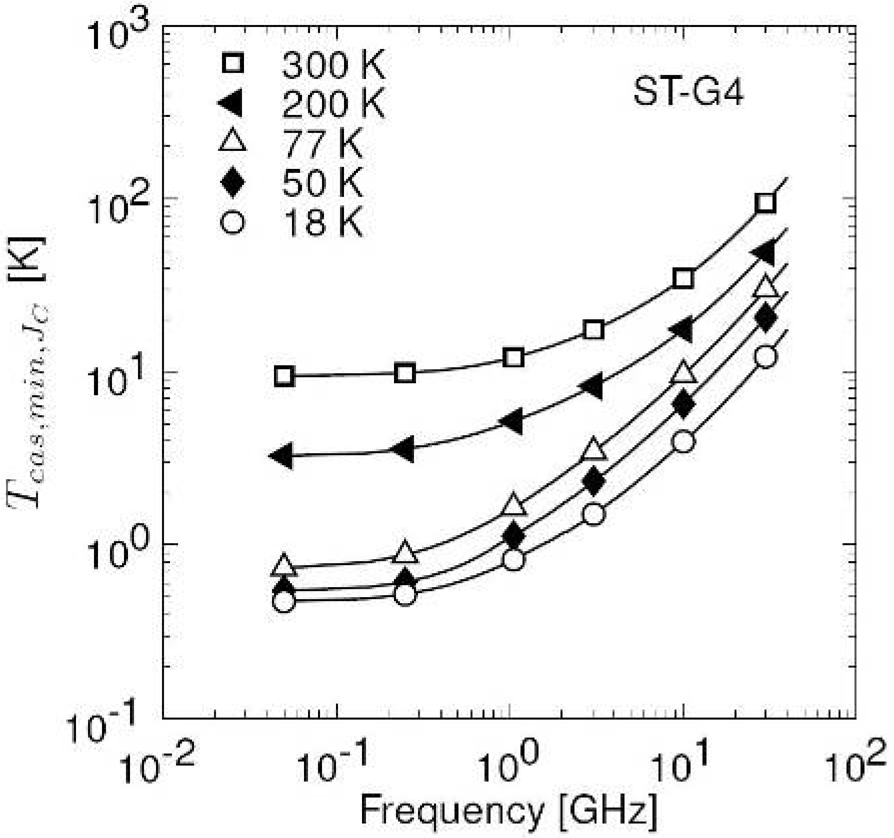
- The correct figure of merit is the noise measure, M, or expressed as a noise temperature, Tcas = M*290K = Tn / (1 – 1/G) where G is the available power gain. It is the noise temperature of an infinite cascade of identical amplifiers. For G >>1, Tcas ~ Tn.

- The golden rule: Tcasmin, the minimum value of Tcas with respect to source impedance, is independent of all lossless network embedding including feedback networks. [Haus, 1958]. As feedback reduces gain, Tmin, is reduced so Tcas is constant.

Tcasmin for a ST BiCMOS9MW SiGe Transistor

From *Silicon-Germanium Heterojunction Bipolar Transistors For Extremely Low-Noise Applications,* J. C. Bardin, Ph.D. thesis, Caltech, June, 2009

- Curves are at optimum current density for each frequency and temperature; see thesis for details and other noise parameters
- At 300K Tcasmin at 1 GHz is ~ 10K and at 77K is ~1K.



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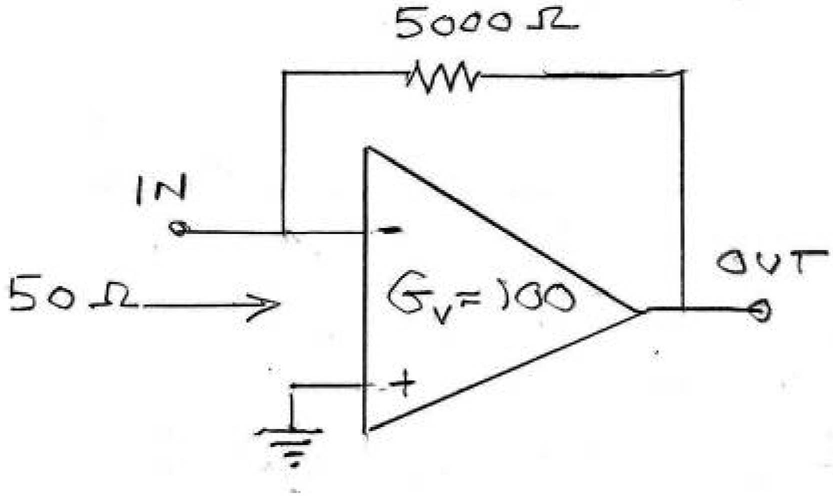
Resistive Feedback, Simple Example

- Reactive feedback can provide simultaneous noise and gain match in narrow band amplifiers. This is usually in the from of inductance in the source or emitter common terminal to ground.

- Resistive feedback can provide gain match in a wideband amplifier with little effect on the noise. In the circuit below 5000 ohms to an output node with voltage gain of -100 provides 50 ohm input impedance at all frequencies. The effect on noise is the same as 5000 ohms from input to ground which contributes a noise of 50/5000 *300K = 3K for the feedback resistor at room temperature.



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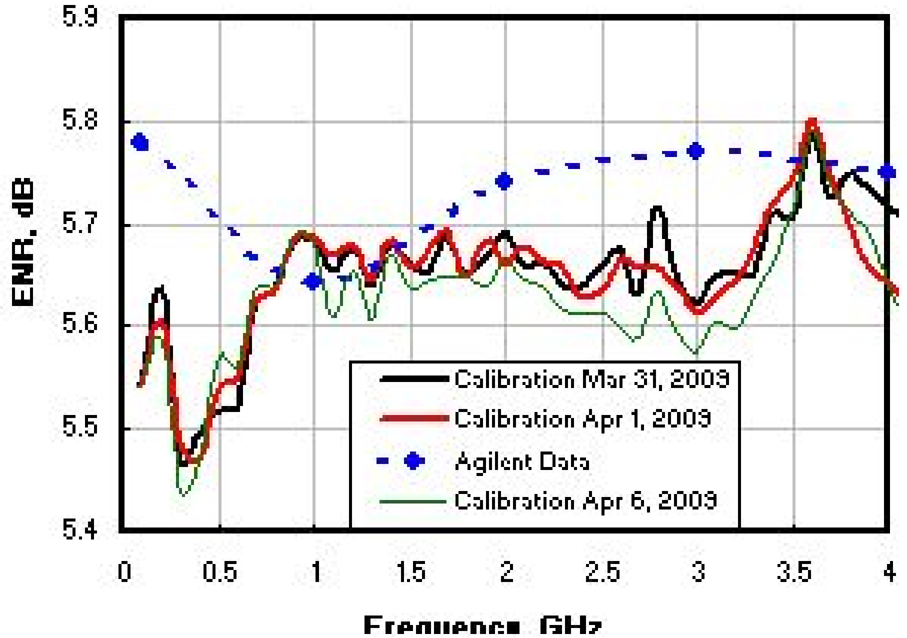
Coaxial 50 Ohm Noise Source Calibration

- The specified uncertainty of the ENR of Agilent noise sources is +/- 0.15dB. For LNA noise temperatures under 100K this results in a a noise temperature uncertainty of +/-10K
- The noise source uncertainty limits the accuracy of transistor manufacturer’s data at low frequencies where the data sheet noise figures are of the order 0.2 dB at 2 GHz. This is 14K +/- 10K

Noise source ENR can be calibrated with LN2 terminations such as the one at right developed at NRAO in 1983. The noise temperature at the SMA connector is believed to be known to +/- 1K. LN2 terminations are also available from Maury Microwave.

Calibration of Two Agilent Noise Sources with NRAO LN7 Noise Standard

by Hamdi Mani, Apr 6, 2009



Top Graph is for N4000A

Bottom graph is for 346A

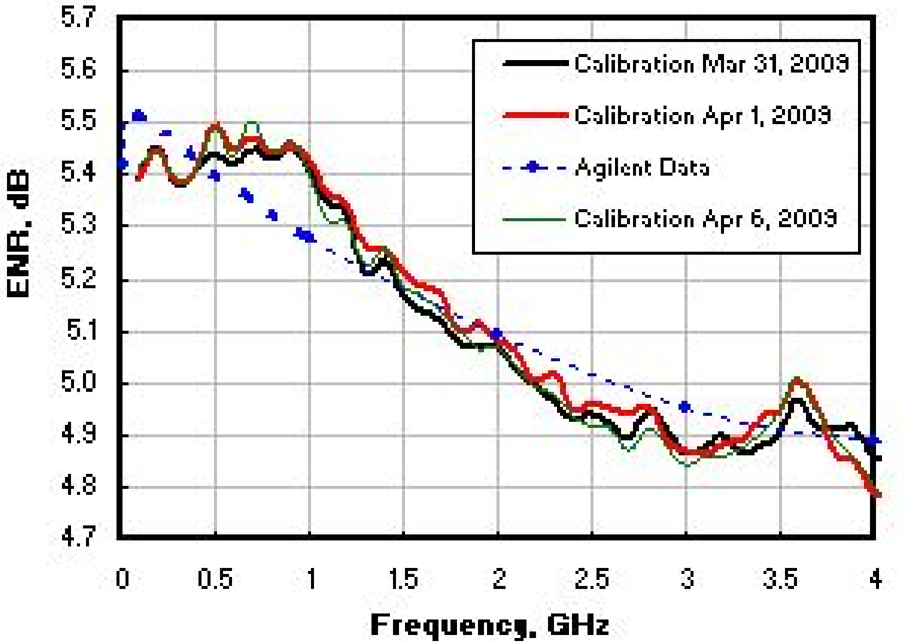
Red, black, and green curves show repeatability within +/- .05dB on 3 days.

Agilent values in blue dashed.

Conclusions:

In the 0.1 to 4 GHz range the N4000A differs from the LN7 by as much as 0.2 dB.

The peak difference for the 346A is 0.15 dB

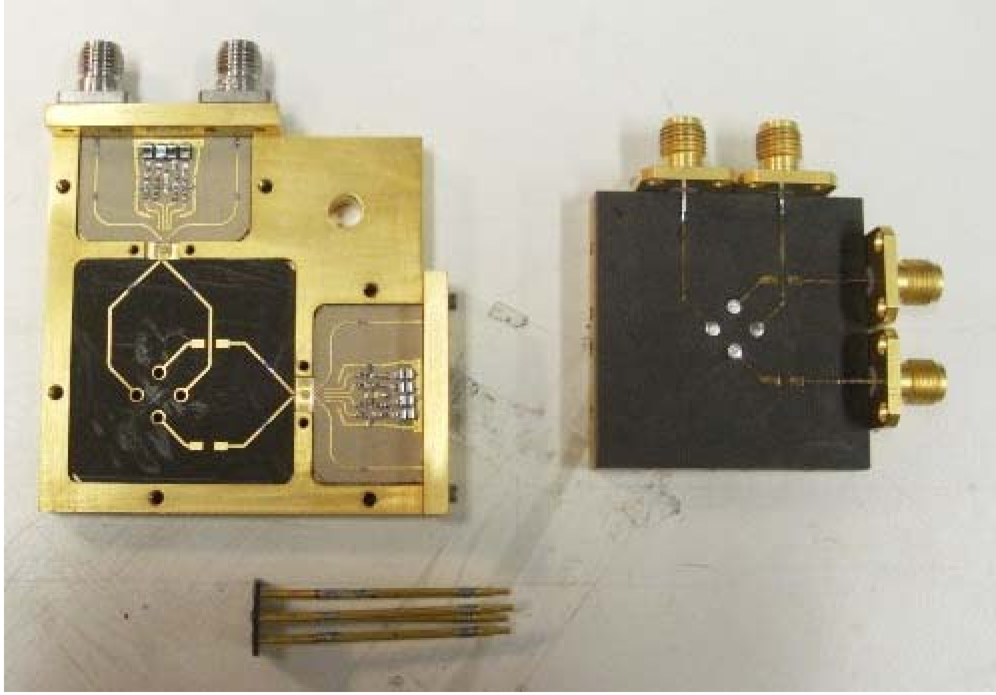


Noise Temperature Measurement of Differential LNA’s

- Noise temperature of a differential amplifier can be measured with the Y factor method by dipping a termination resistor in LN2 at 77K

- The resistance change vs temperature can be measured very accurately at DC with on ohmmeter. The change in typical thin-film resistance is negligible.

- Shown below is a small board with two 270 ohm resistors connected to two differential LNA’s with a gold-plated SS tubing quad transmission lines.



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Noise Parameters from S Parameters

- Noise parameters can be determined by measuring the noise temperature at several generator impedances. For very low noise transistors this is not easily done with sufficient accuracy especially if the Zopt for the transistor is far from 50 ohms and the noise parameters at cryogenic temperatures are desired.

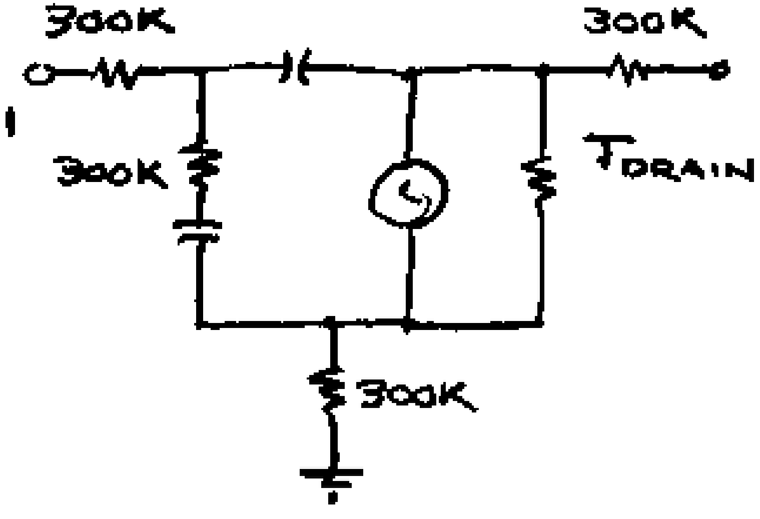
- An alternative is to measure the S parameters and DC characteristics of the transistor as a function of frequency and find the equivalent circuit of the transistor from these S parameters. Then transistor device theory is applied to reduce the number of unknown noise variables to 2, 1, or 0 frequency-independent numbers. The number depends upon the type of device (i.e FET or bipolar) and required accuracy.

- Noise parameters of any linear passive network can be calculated from the S parameters and temperature of the network. Thus noise due to input and other circuits in a low noise amplifiers can be accounted for.

- All of the above calculations can be most easily performed with a circuit simulator such as Microwave Office.

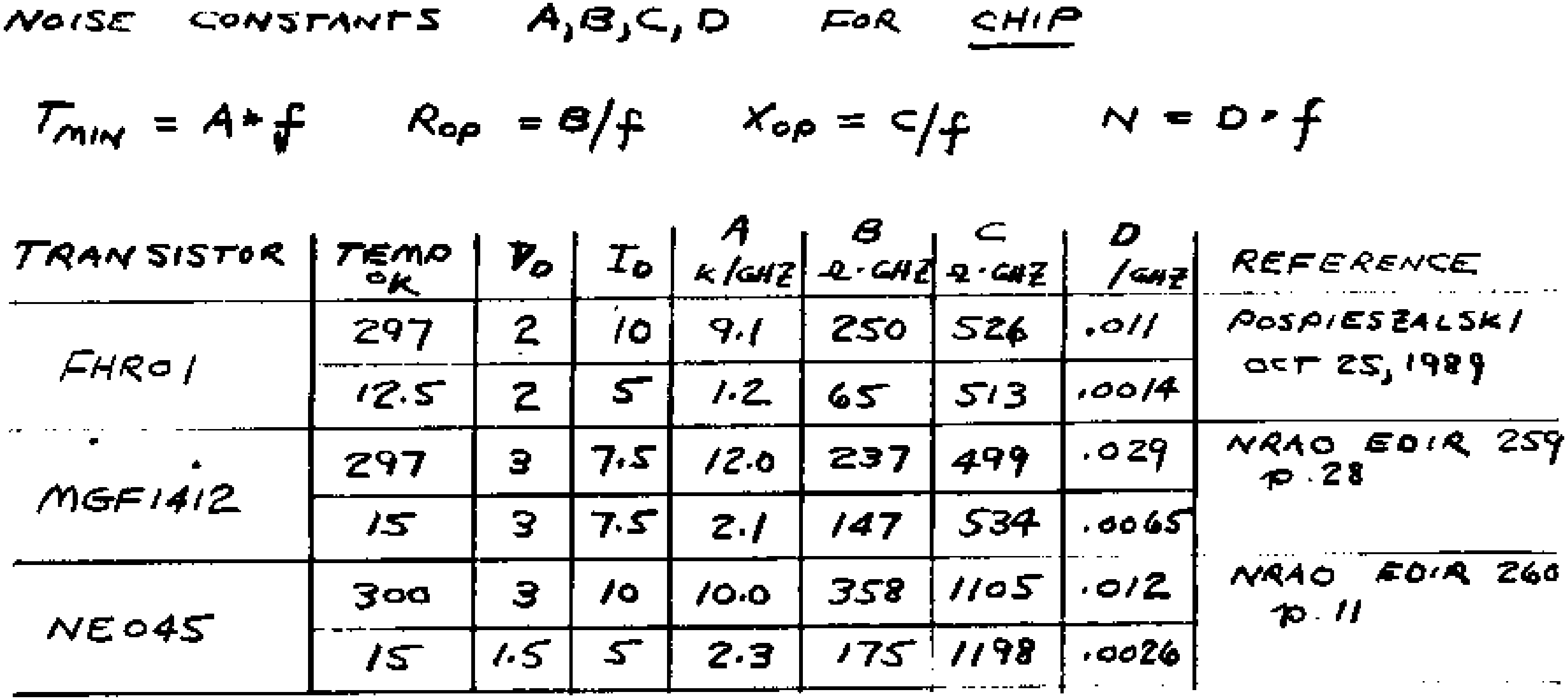
Noise Parameters of a FET or HEMT

- Procedures exist to find the equivalent circuit from measurements of the S parameters vs frequency. The elements in the equivalent circuit have a physical basis; they can be identified with regions of the transistor. Regions with resistive loss generate thermal noise with noise power proportional to physical temperature.
- As suggested by Pospieszalski [1989] the noise due to turbulence in DC current flow within the transistor channel can be modeled by assigning a temperature, Tdrain, to the shunt drain resistor determined by the model..
- Tdrain is independent of frequency and thus one measurement of noise temperature at one frequency can be used for find Tdrain. Then all 4 noise parameters as a function of frequency can be calculated.



The Noise Parameters of a FET Chip Have Simple Frequency Dependence

- Linear dependence upon frequency used to extrapolate noise parameters of commercial GaAs FETs many years ago.
- The linear dependence of Tmin allows a Tmin measurement at a higher frequency (say 4 GHz or 12 GHz) to be extrapolated to a lower frequency where the Tmin is too small to be accurately measured.



NOISE CONSTANTS A, B, C, D FOR CHIP

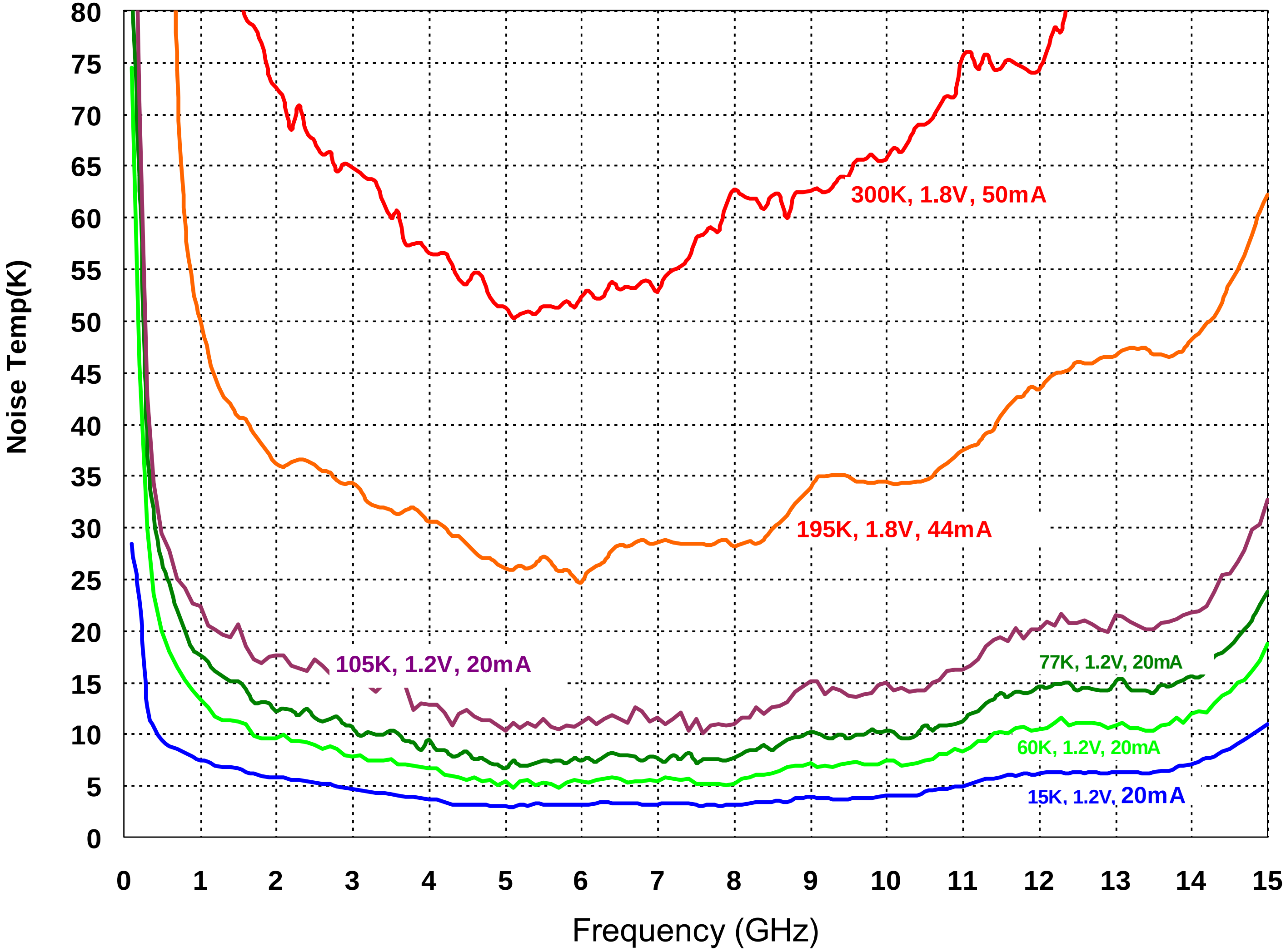
$T_{MIN} = A * f$ $R_{op} = B/f$ $X_{op} = C/f$ $N = D \cdot f$

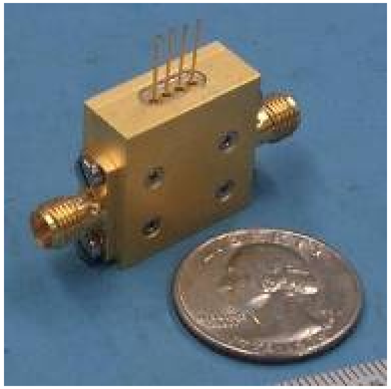
TRANSISTOR	TEMP °K	V_D	I_D	A K/GHZ	B Ω·GHZ	C Ω·GHZ	D /GHZ	REFERENCE
FHR01	297	2	10	9.1	250	526	.011	POSPIESZALSKI OCT 25, 1989
	12.5	2	5	1.2	65	513	.0014	
MGF1412	297	3	7.5	12.0	237	499	.029	NRAO EDIR 259 P. 28
	15	3	7.5	2.1	147	534	.0065	
NE045	300	3	10	10.0	358	1105	.012	NRAO EDIR 260 P. 11
	15	1.5	5	2.3	175	1198	.0026	

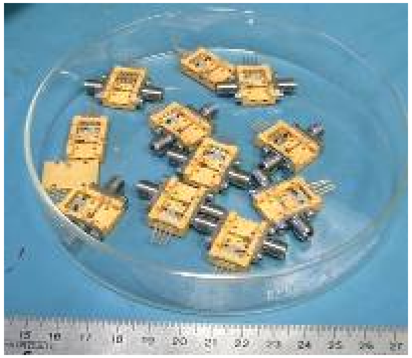
Noise Temperature vs Frequency at 300K, 195K, 105K, 77K, 60K, and 15K

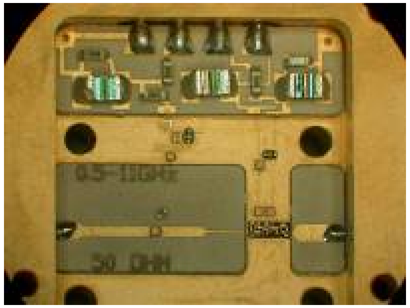
InP HEMT MMIC, WBA13, Tested at Caltech May, 2007

Over 300 of these modules in use in radio astronomy and physics research.





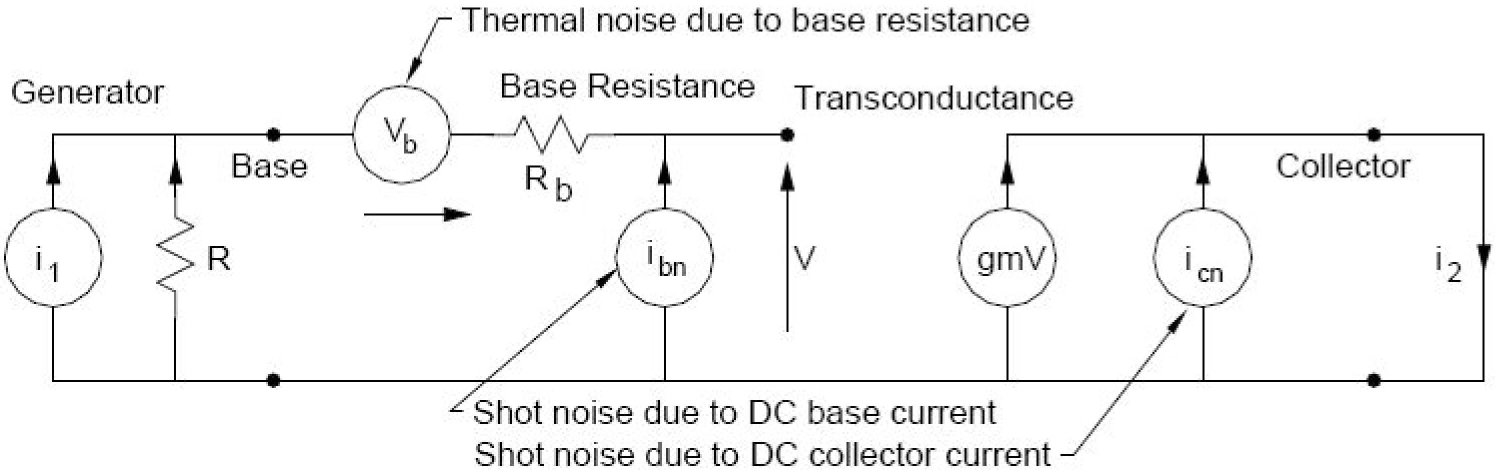




Noise of SiGe Bipolar Transistors

- Noise in SiGe transistor consists of thermal noise plus shot noise due to base and collector currents. The shot noise power is proportional to the DC base and collector currents and is thus determined by DC measurements.

- A simple model which gives Tmin ~ 290K / √β where β is the DC current gain. This approximation holds for all temperatures at frequencies below a few GHz. [See Weinreb and Bardin, Nov 2007, IEEE MTT Transactions]

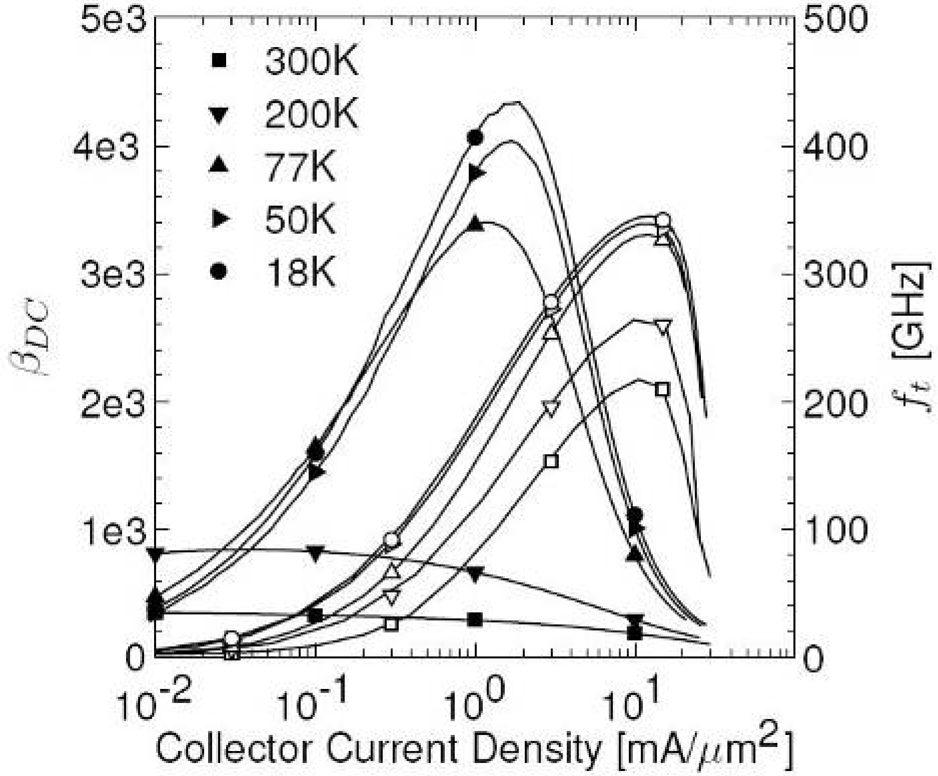


- A complete noise theory is given in the J. Bardin Ph.D. thesis, Caltech, 2009.

SiGe Transistors Greatly Improve with Cooling

- Example below shows current gain (left) and Ft (right) for an IBM 8HP transistor as a function of collector current density at temperatures from 300K to 18K

- The current gain improves from 300 to 4300 and the Ft increases from 200 GHz to 340 GHz.



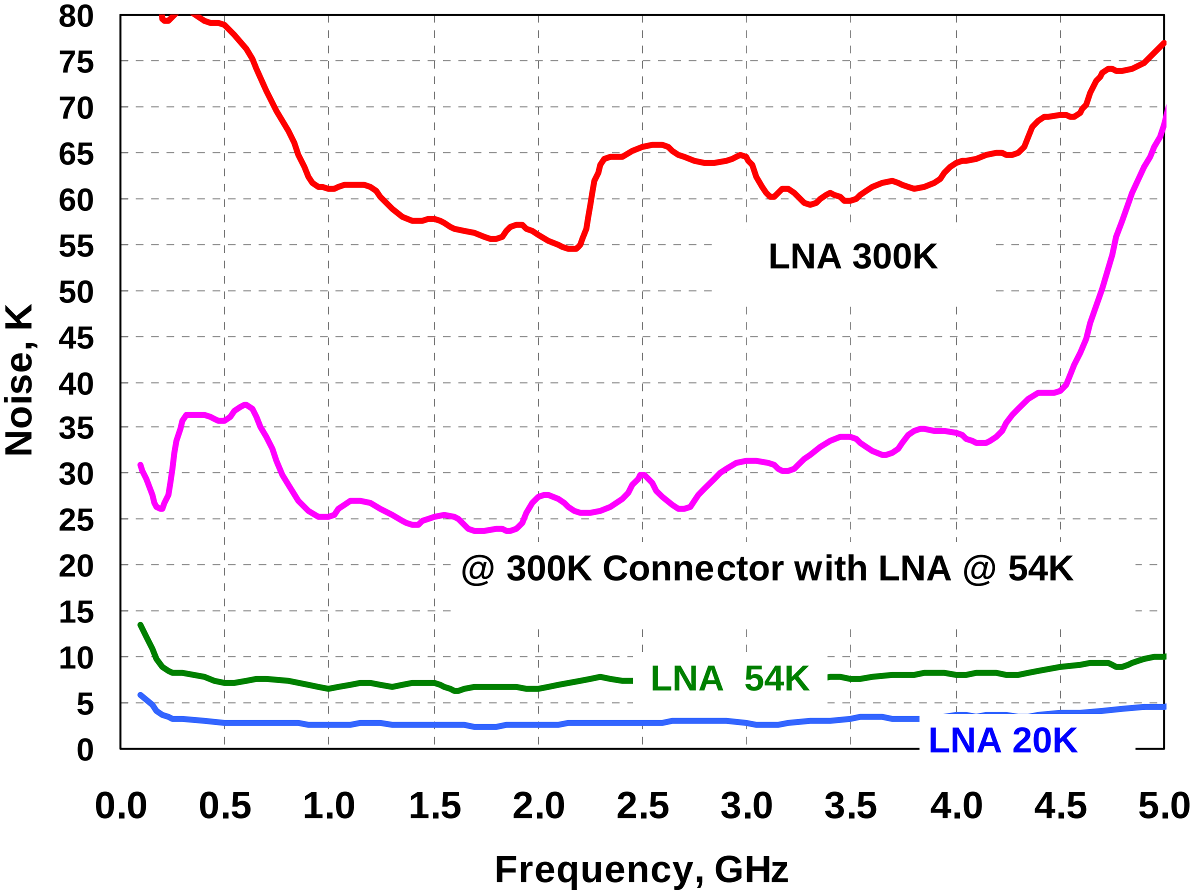
(a) IBM-G4

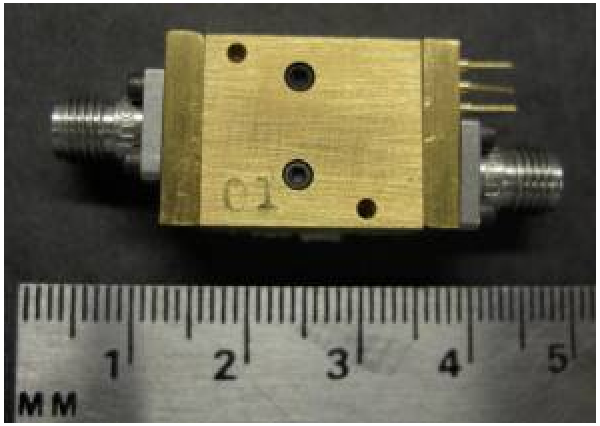
, Jun 2009

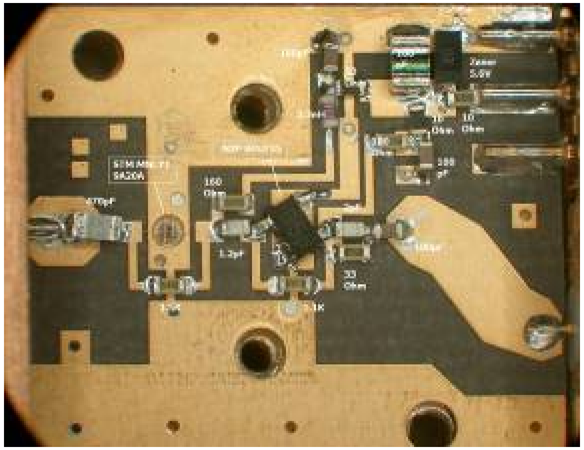
Noise vs Frequency of SiGe Transistor LNA at 3 Temperatures

ST first stage, NXP 2nd stage, tested May, 2008

Typical gain 35 dB, typical bias 2V, 12mA

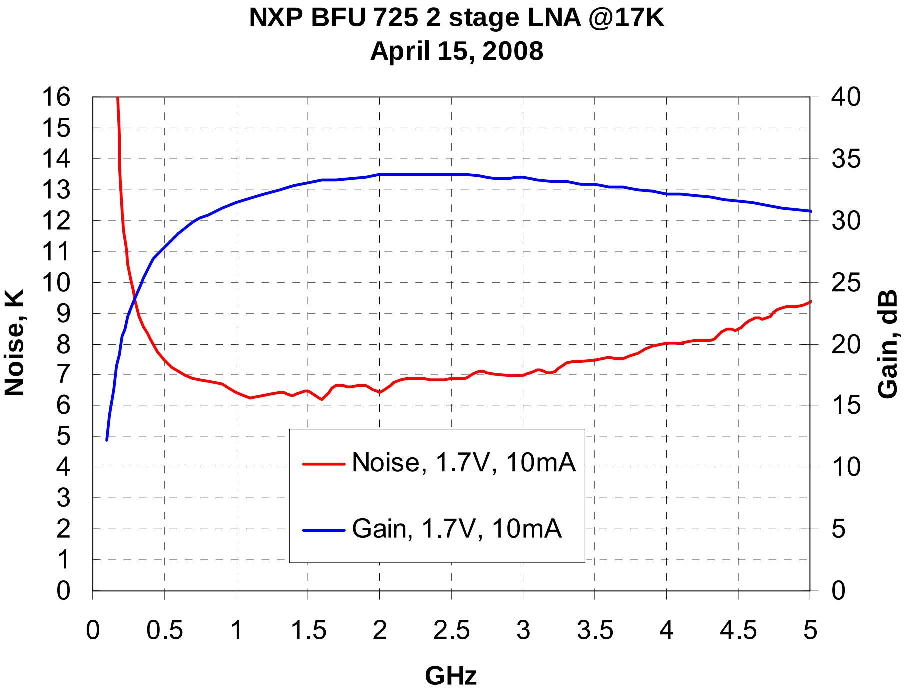




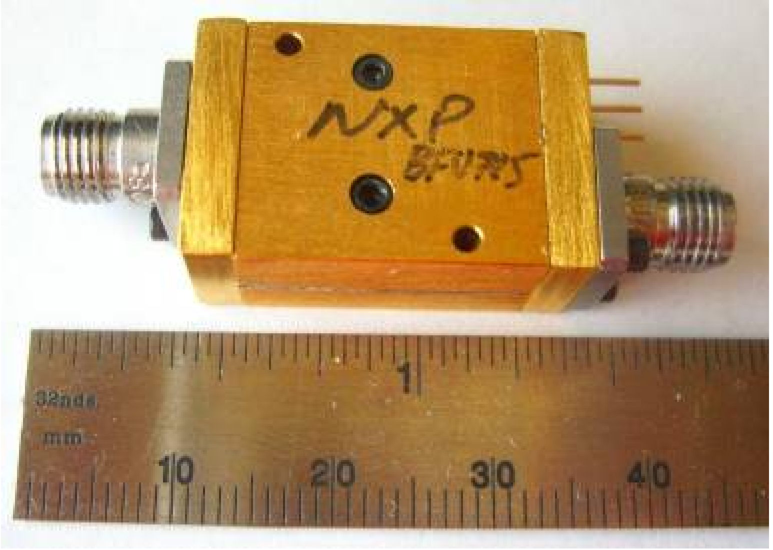


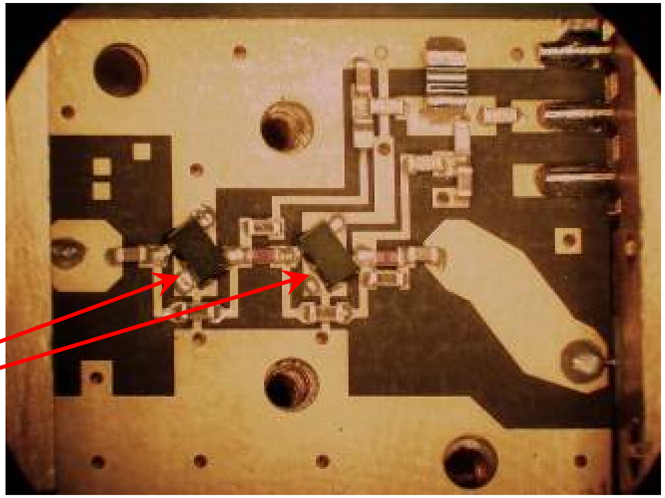
Low-Cost SiGe 0.5 to 4 GHz Cryogenic LNA

- 7K noise at 17K with $.44 NXP transistor
- With STM transistor input stage noise is 2.5K at 17K, and 7K at 55K.



NXP BFU 725 2 stage LNA @17K
April 15, 2008





SiGe transistors in 2mm plastic package on printed circuit board

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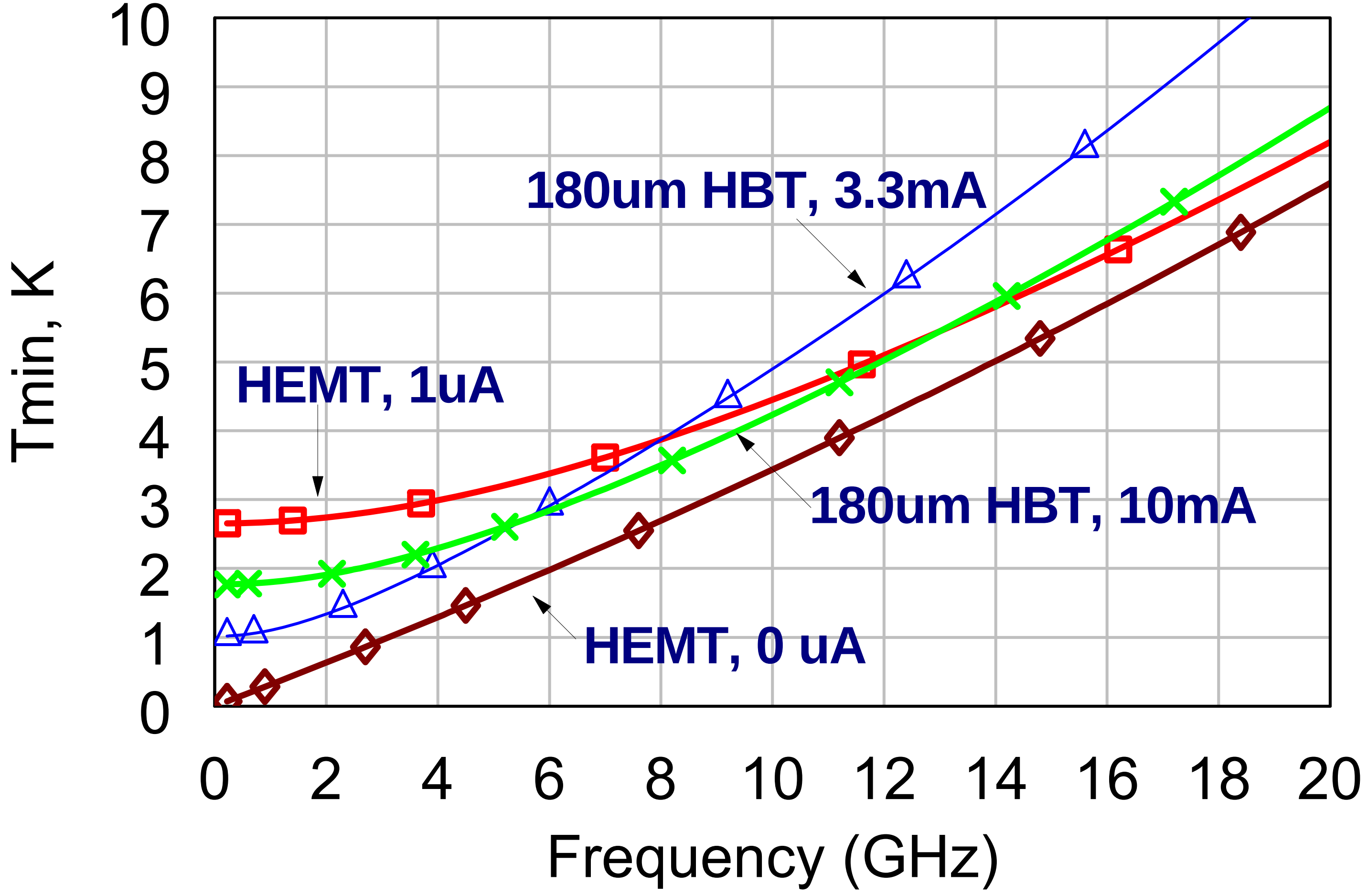
SiGe HBT and InP HEMT Minimum Noise at 15K

Results below are modeled. As a confirmation of the model an HBT single-stage cascode amplifier has been measured with 2K noise temperature and 28 dB gain at 1 GHz.



With Cooled LNA's Most of the System Noise is Not in the LNA

- Assume feed for 0.3 to 1.4 GHz is at 300K.
- Feed for 1.4 to 10 GHz must be cooled or partially cooled.
- No calibration signal coupling is assumed.

Component	**Remarks**	**2009, K LNA at 300K**	**2009, K LNA at 60K**	**2011, K LNA at 300K**	**2011, K LNA at 60K**
Sky	Background + atmosphere	4	4	4	4
Spillover & Blockage	Reduce with offset antenna	12	12	6	6
Feed loss	Estimate, measure by 2010	7	7	5	5
LNA to feed loss	10cm of 0.141 Cu coax, .04 dB at 300K	4	4	3	3
Vacuum feedthru	Glass/Kovar bead, 0.1 dB	0	7	0	5
Coax in dewar	10cm or .141 SS/BeCu .09 dB at 190K	0	4	0	3
LNA	Robust, differential LNA measured at connector	40	12	25	5
Total	**Estimate, +/- 5K**	**67**	**50**	**43**	**31**

REASONS FOR LARGE ERRORS IN MM WAVE TRANSISTOR NOISE MEASUREMENTS

1) COMPARABLE CIRCUIT AND DEVICE NOISE

2) 300K GENERATOR NOISE » DEVICE NOISE

3) DEVICE HAS LOW GAIN AND SECOND STAGE NOISE IS HIGH

4) LOW FREQUENCY OSCILLATION

5) CHIP MEASUREMENT PLANE POORLY DEFINED

6) OPTIMUM NOISE IMPEDANCE IS FAR FROM 50 OHMS

7) UNKNOWN NOISE FROM SEMICONDUCTOR VARIABLE TUNERS

Frequency Asked Question, FAQ1

If a low-noise amplifier has a poor input power match, i.e. high VSWR and low input return loss, will this degrade the degrade the system noise figure?

Short Answer, SA1

No, NF and input power match are not directly related. If the amplifier has a low noise figure with a 50 ohm source impedance it will give a low system noise figure even if it has a high input VSWR.

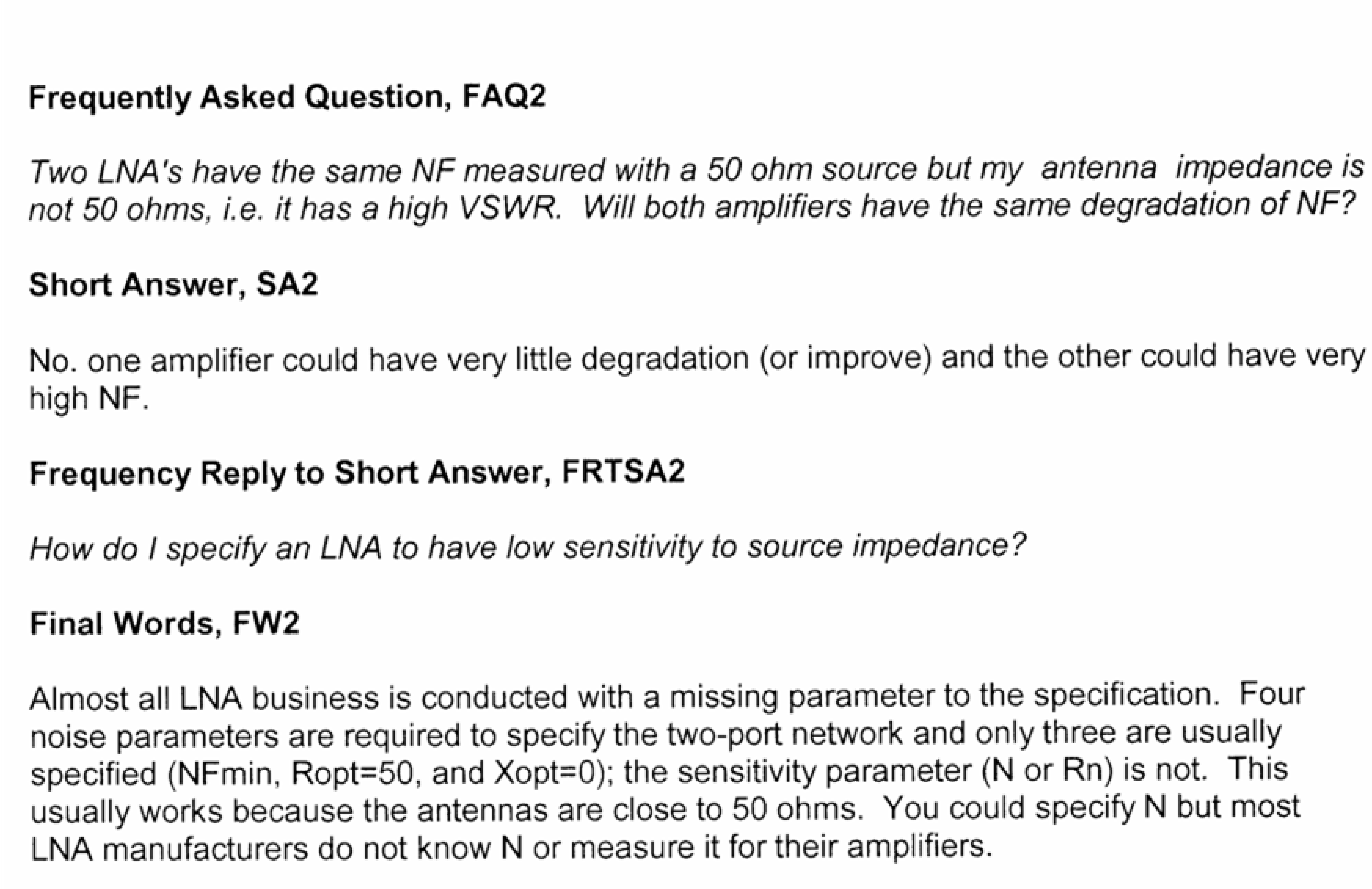
Frequent Reply to Short Answer, FRTSA1

But if most of the input signal is being reflected by the amplifier and NF is a measure of the noise-to-signal ratio, couldn't you should get a better NF by matching more of the signal into the amplifier.

Final Words, FW1

Yes, you can get more signal power into the amplifier by matching but the noise power going into the amplifier will also increase in such a way that the nose-to-signal ratio and NF are degraded.

The penalty of high input VSWR is a large variation of gain as the source impedance varies. If a long cable connects the source (antenna) and LNA then there will be a variation of gain (ripple) with frequency. See the following view graph.



Frequently Asked Question, FAQ2

Two LNA's have the same NF measured with a 50 ohm source but my antenna impedance is not 50 ohms, i.e. it has a high VSWR. Will both amplifiers have the same degradation of NF?

Short Answer, SA2

No. one amplifier could have very little degradation (or improve) and the other could have very high NF.

Frequency Reply to Short Answer, FRTSA2

How do I specify an LNA to have low sensitivity to source impedance?

Final Words, FW2

Almost all LNA business is conducted with a missing parameter to the specification. Four noise parameters are required to specify the two-port network and only three are usually specified (NFmin, Ropt=50, and Xopt=0); the sensitivity parameter (N or Rn) is not. This usually works because the antennas are close to 50 ohms. You could specify N but most LNA manufacturers do not know N or measure it for their amplifiers.

Frequently Asked Question, FAQ3

You sold me an amplifier that you measured at 2 dB NF and yet, when I connect it to my antenna, the system NF appears to be 10 dB. My antenna impedance is exactly 50 ohms at the frequency of interest. What is going on here?

Short Answer, SA3

The antenna impedance is the same at the operating frequency but is probably not the same at out-of-band frequencies as the impedance of my amplifier test set-up. The antenna impedance at out-of-band frequencies causes the LNA to oscillate, a large-signal non-linear phenomenon, which degrades the NF at the operating frequency.

Frequent Reply to Short Answer, FRTSA3

^++[@$!!!!

Final Words, FW3

Yes, I better make you a better amplifier! All LNA's should be stable for any passive out-of-band source and load impedances. This can be tested by connecting sliding short circuits to the input and output and observing the bias current (effected by oscillation) or the output signal with a spectrum analyzer.

Frequently Asked Question, FAQ4

Can the noise figure of an amplifier be reduced by feedback?

Short Answer, SA4

Yes, the noise figure of <u>any</u> amplifier can be reduce to 0 dB with a feedback network.

Frequent Reply to Short Answer, FRTSA4

Wha-at? How can that be? Why do we need 0.1μm endodomorphic, super-charged feel-effected transistors?

Final Words, FW4

The proof is a feedback network with a pair of wires from output to input and no coupling to the noisy amplifier. This will have 0 dB noise figure. The problem is that noise figure is not the fundamental noise quality factor of an amplifier - it is, only if the amplifier has high gain. The fundamental quality factor is noise measure which is invariant to lossless feedback (and degraded by lossy feedback). As negative feedback is added to an amplifier, both gain and NF are reduced in such a way that the NF of a infinite cascade of the amplifiers is unchanged.

Frequently Asked Question, FAQ6

Should I use a hybrid integrated circuit (MIC) or monolithic integrated circuit (MMIC) LNA in my system?

Short Answer, SA6

None, it is not even clear which technology is less costly for large quantities. The performance is similar if the same transistors are used and the MMIC has been correctly designed or optimized by repeated design revisions.

Answer

Factors favoring an MIC approach

1) Quantities less than 1000
2) Frequencies less than 30 GHz
3) Required semiconductor area is smaller - just a 0.4mm x 0.4mm transistor chip.
4) Individual tuning is feasible.

Factors favoring a MMIC approach

1) Less assembly labor especially if multiple functions are integrated on one chip.
2) Precise and repeatable circuits defined to sub-micron precision by lithography.
3) Very small and accurate circuit elements are feasible.
4) Wide bandwidth because of lower parasitic reactances.
5) Very small, complex arrays are feasible.

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