

# Spatial RFI mitigation

## *Concepts, performances and implementation*

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# Outline

1. Data model & Radio interferometry
2. Spatial filtering – oblique projection
3. RFI subspace estimation
4. Implementation – tests with Embrace
5. Conclusion

# Data model & Radio interferometry



=



+



+



Radio  
telescope

=

Cosmic  
source

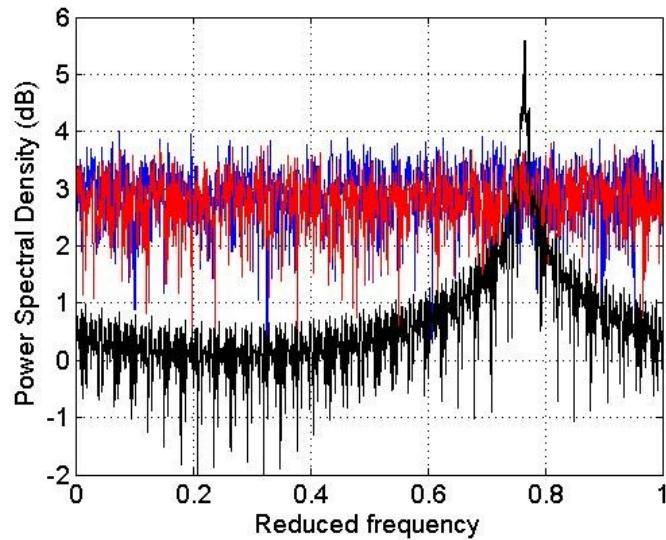
+

Interference

+

System  
noise

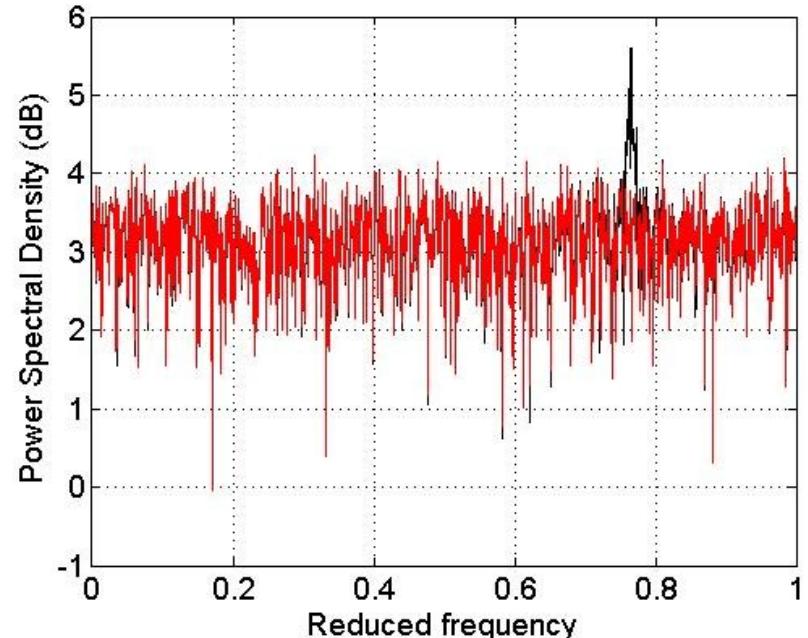
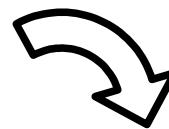
# Data model & Radio interferometry



Cosmic source

Noise

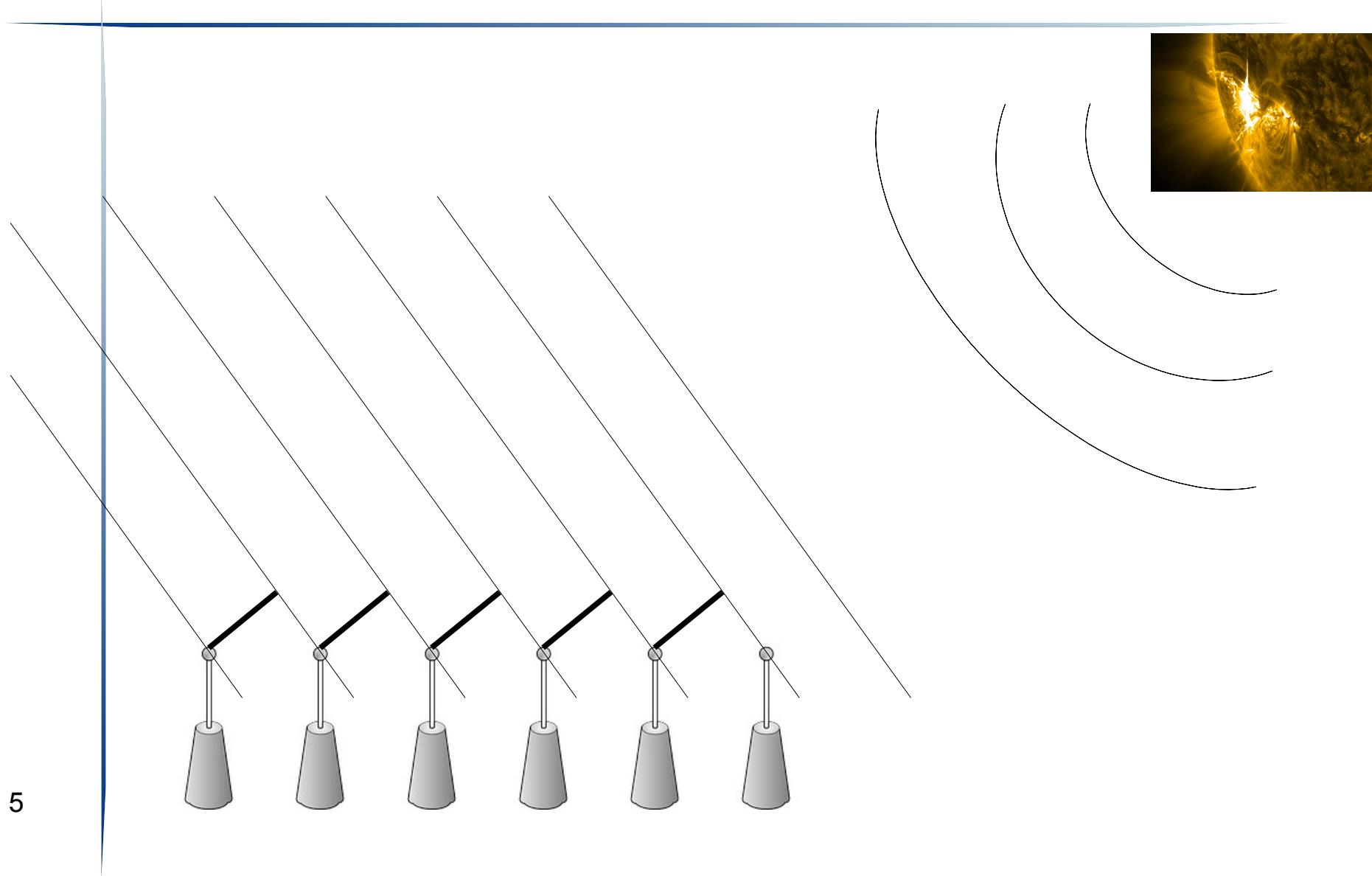
RFI



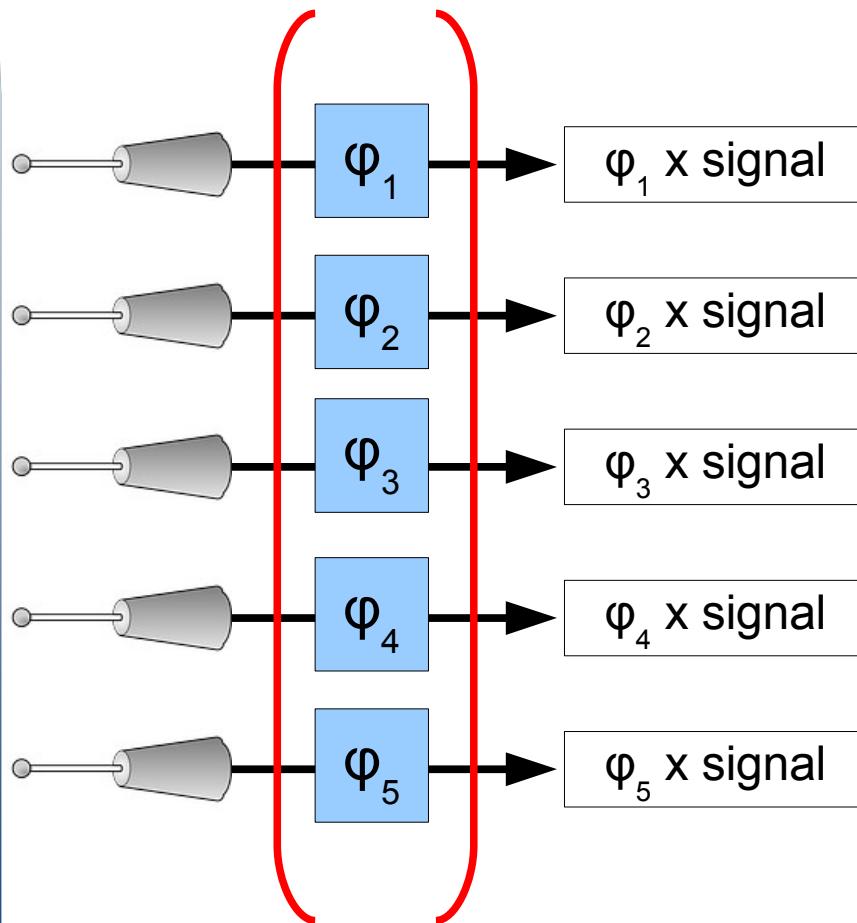
Total spectrum

Total spectrum without RFI

# Data model & Radio interferometry



# Data model & Radio interferometry



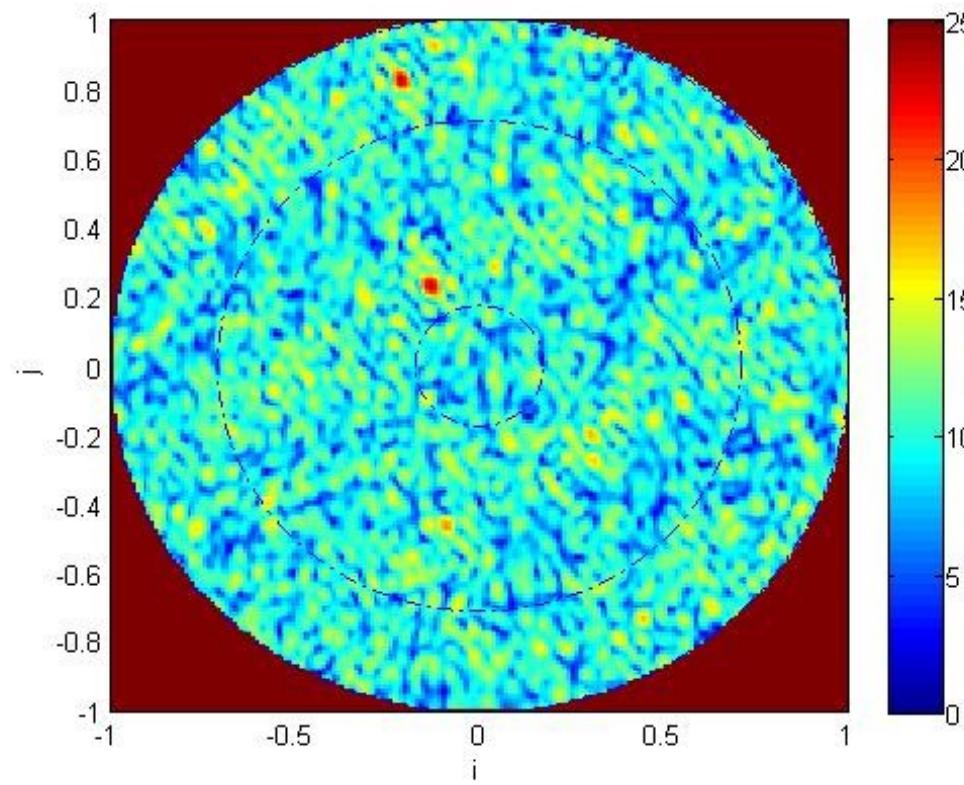
$$\rightarrow x(t) = As * \text{signal}(t)$$

Observation:

$$x(t) = \textcolor{red}{As}_1 * \text{source}_1(t) + \\ \textcolor{red}{As}_2 * \text{source}_2(t) + \\ \textcolor{red}{As}_3 * \text{source}_3(t) + \\ \dots + \\ \textcolor{red}{As}_N * \text{source}_N(t)$$

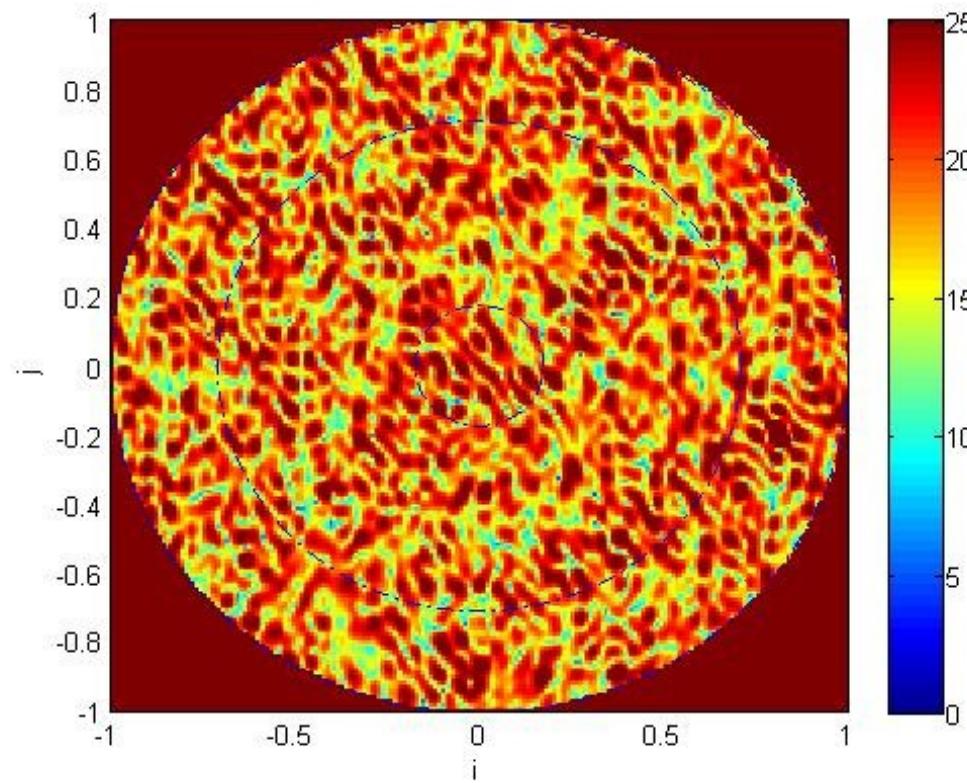
# Data model & Radio interferometry

Without interference



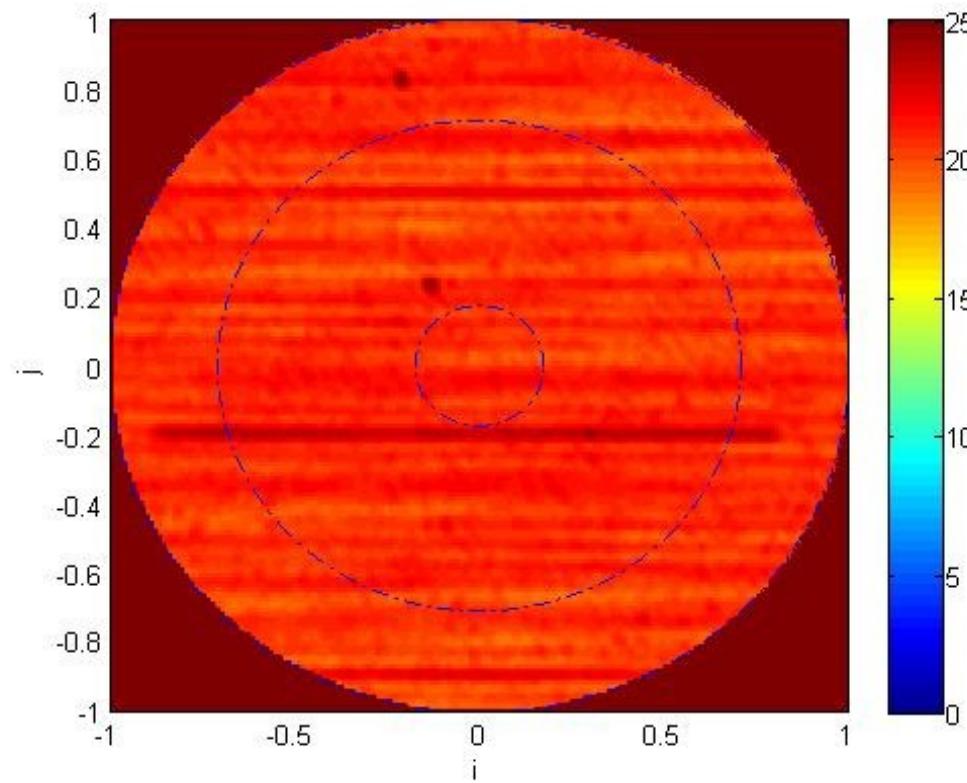
# Data model & Radio interferometry

With a strong interference



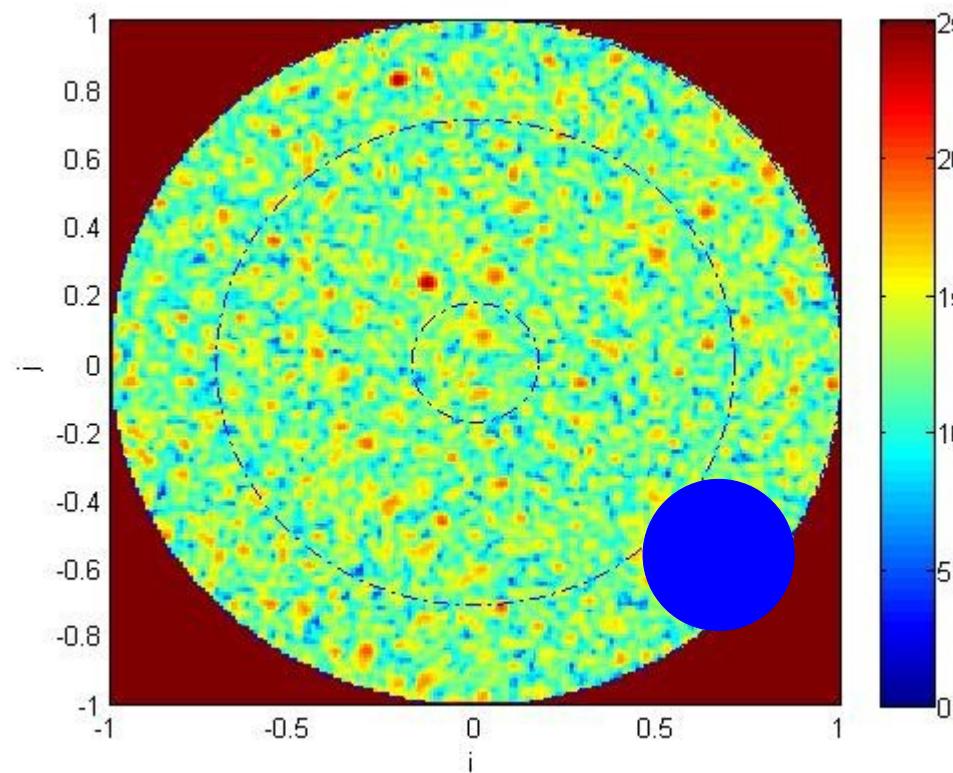
# Data model & Radio interferometry

With a moving interference



# Spatial filtering

Basic idea:

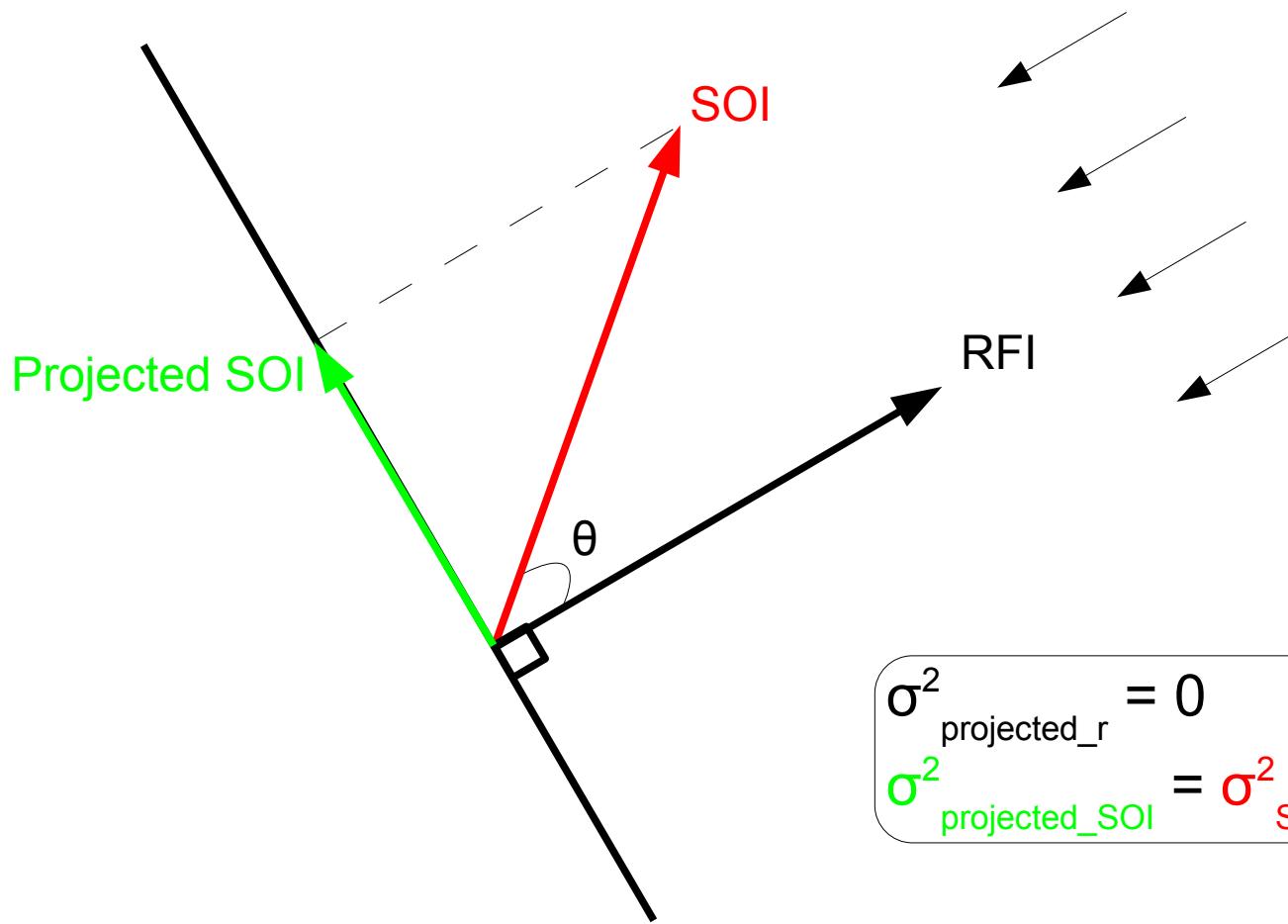


10

→ recover the cosmic source-of-interest power

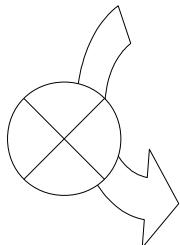
# Spatial filtering

## Orthogonal projector



# Spatial filtering

## Orthogonal projector



$$\mathbf{P} = \mathbf{I} - \mathbf{A}_r \cdot (\mathbf{A}_r^H \cdot \mathbf{A}_r)^{-1} \cdot \mathbf{A}_r^H$$

$$\mathbf{x}(t) = \mathbf{A}_s * \text{SOI}(t) + \mathbf{A}_r * \text{RFI}(t) + \text{noise}(t)$$

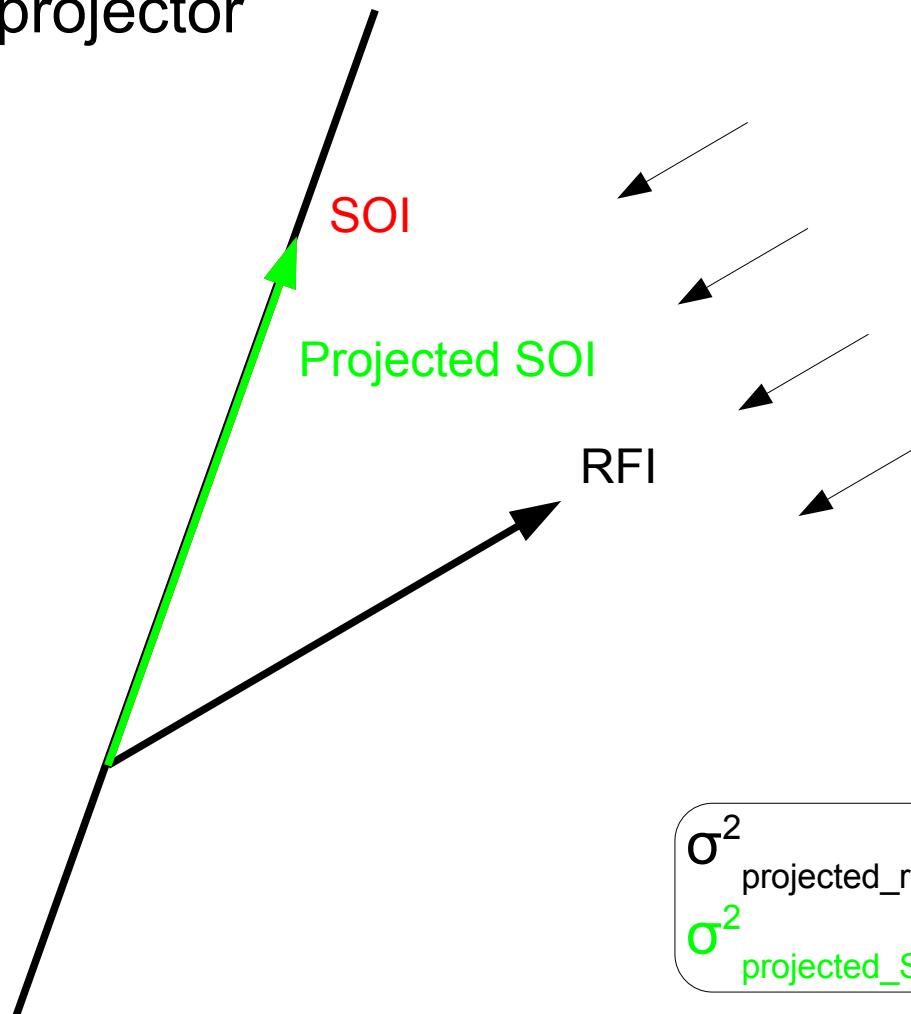
$$\mathbf{P} \cdot \mathbf{A}_r \cdot \text{RFI}(t) = 0$$

$$\mathbf{P} \cdot \mathbf{A}_s \cdot \text{SOI}(t) = (\mathbf{A}_s - \mathbf{A}_r \cdot (\mathbf{A}_r^H \cdot \mathbf{A}_s)) \cdot \text{SOI}(t)$$

$$\mathbf{P} \cdot \text{noise}(t) = (\mathbf{I} - \mathbf{A}_r \cdot \mathbf{A}_r^H) \cdot \text{noise}(t)$$

# Spatial filtering

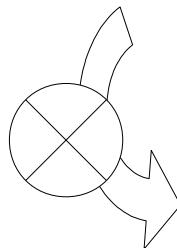
→ Oblique projector



$$\sigma_{\text{projected\_r}}^2 = 0$$
$$\sigma_{\text{projected\_SOI}}^2 = \sigma_{\text{SOI}}^2$$

# Spatial filtering

## Oblique projector



{

$$\mathbf{E} = \mathbf{A}_S \cdot (\mathbf{A}_S^H \cdot \mathbf{P} \cdot \mathbf{A}_S)^{-1} \cdot \mathbf{A}_S^H \cdot \mathbf{P}$$

$$\text{with } \mathbf{P} = \mathbf{I} - \mathbf{A}_R \cdot (\mathbf{A}_R^H \cdot \mathbf{A}_R)^{-1} \cdot \mathbf{A}_R^H$$

$$\mathbf{x}(t) = \mathbf{A}_S * \text{SOI}(t) + \mathbf{A}_R * \text{RFI}(t) + \text{noise}(t)$$

$$\mathbf{E} \cdot \mathbf{A}_R \cdot \text{RFI}(t) = 0$$

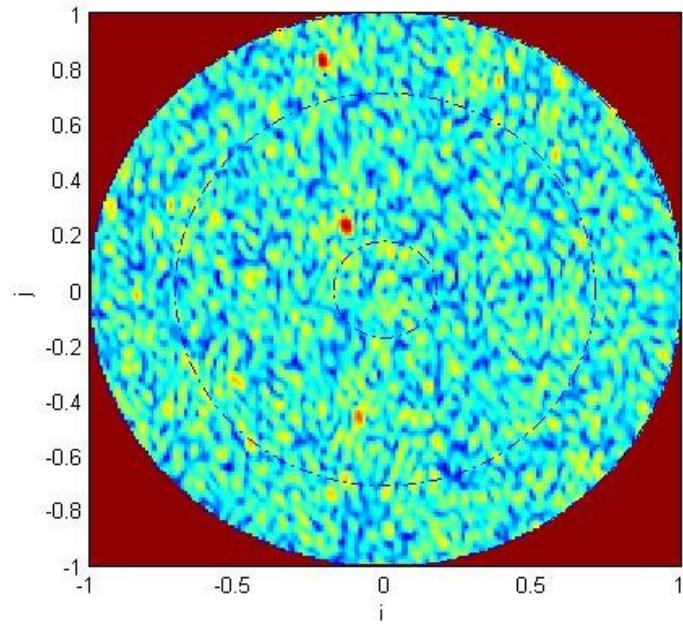
$$\mathbf{E} \cdot \mathbf{A}_S \cdot \text{SOI}(t) = \mathbf{A}_S \cdot \text{SOI}(t)$$

$$\mathbf{E} \cdot \text{noise}(t) = \mathbf{E} \cdot \text{noise}(t)$$

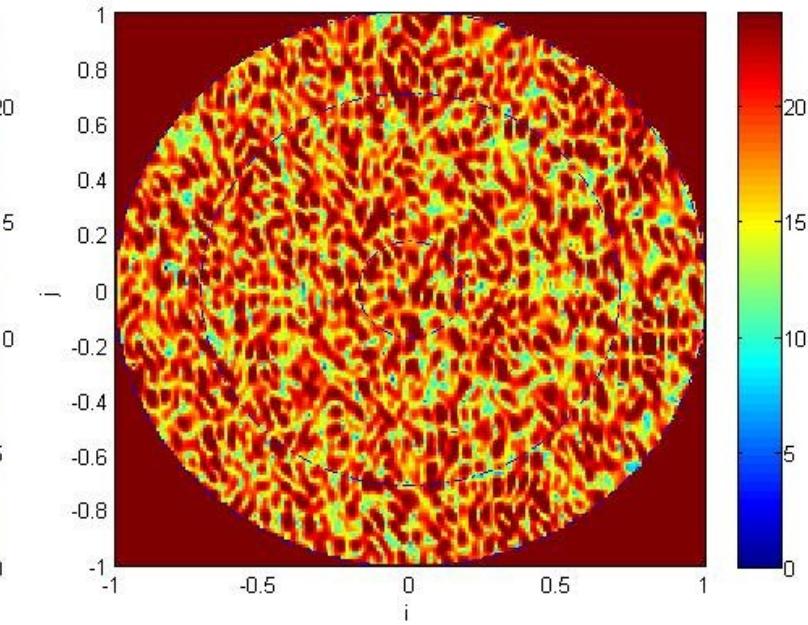
# Spatial filtering

## Oblique projector

Without RFI



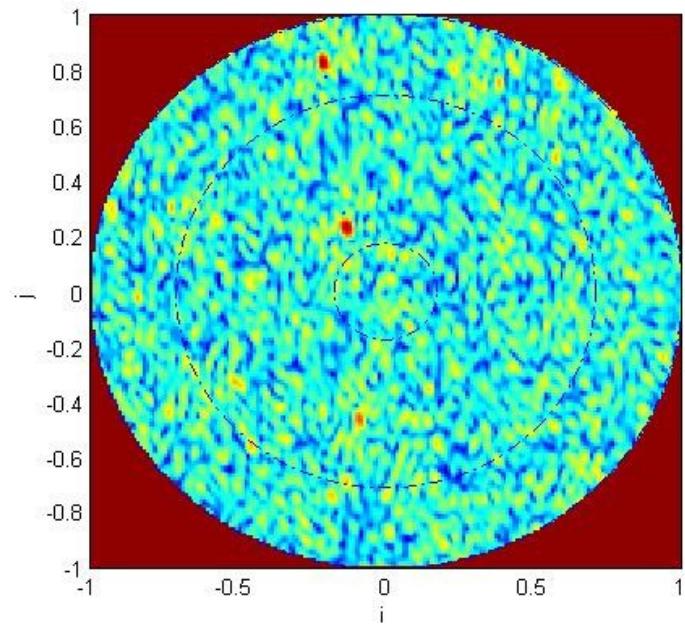
With RFI



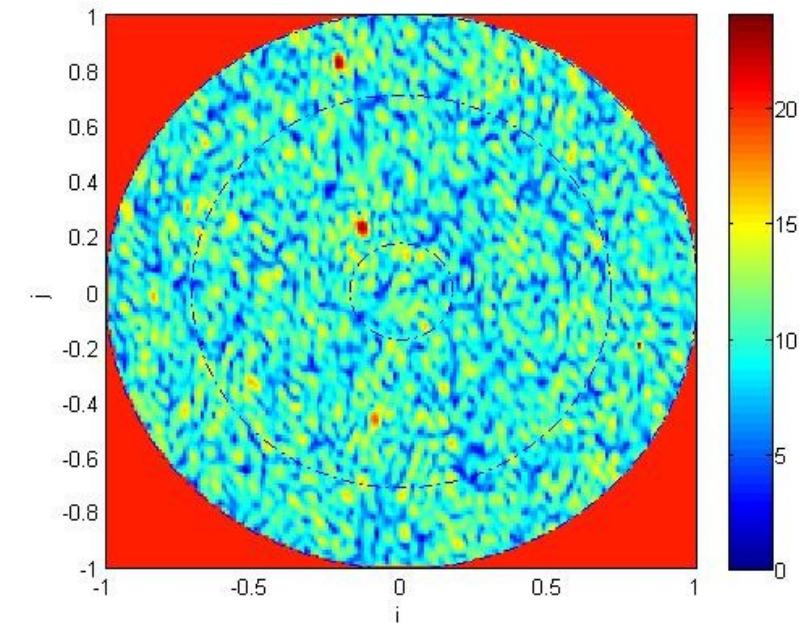
# Spatial filtering

## Oblique projector

Without RFI



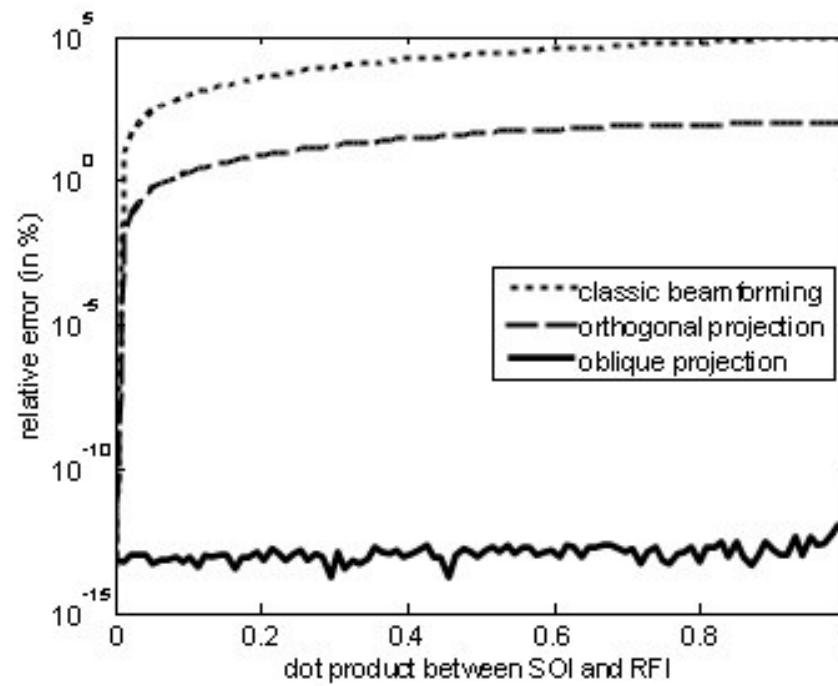
With RFI



After processing

# Spatial filtering

## Orthogonal vs. Oblique projector



# RFI subspace estimation

$$\mathbf{x}(t) = \mathbf{A}_s * \text{SOI}(t) + \mathbf{A}_r * \text{RFI}(t) + \text{bruit}(t)$$

$$\begin{aligned} \mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\} &= E\{ \text{SOI}^2(t) \cdot \mathbf{A}_s^H \cdot \mathbf{A}_s + \\ &\quad \text{RFI}^2(t) \cdot \mathbf{A}_r^H \cdot \mathbf{A}_r + \\ &\quad \text{noise}(t)^H \cdot \text{noise}(t) + \\ &\quad \cancel{\text{SOI}(t)^H \cdot \text{RFI}(t) \cdot \mathbf{A}_s^H \cdot \mathbf{A}_r} + \\ &\quad \cancel{\text{RFI}(t)^H \cdot \text{SOI}(t) \cdot \mathbf{A}_r^H \cdot \mathbf{A}_s} + \\ &\quad \cancel{\text{noise}(t)^H \cdot \text{RFI}(t) \cdot \mathbf{A}_r} + \\ &\quad \cancel{\text{RFI}(t)^H \cdot \mathbf{A}_r^H \cdot \text{noise}(t)} + \\ &\quad \cancel{\text{SOI}(t)^H \cdot \mathbf{A}_s^H \cdot \text{noise}(t)} + \\ &\quad \cancel{\text{noise}(t)^H \cdot \text{SOI}(t) \cdot \mathbf{A}_s} \} \end{aligned}$$

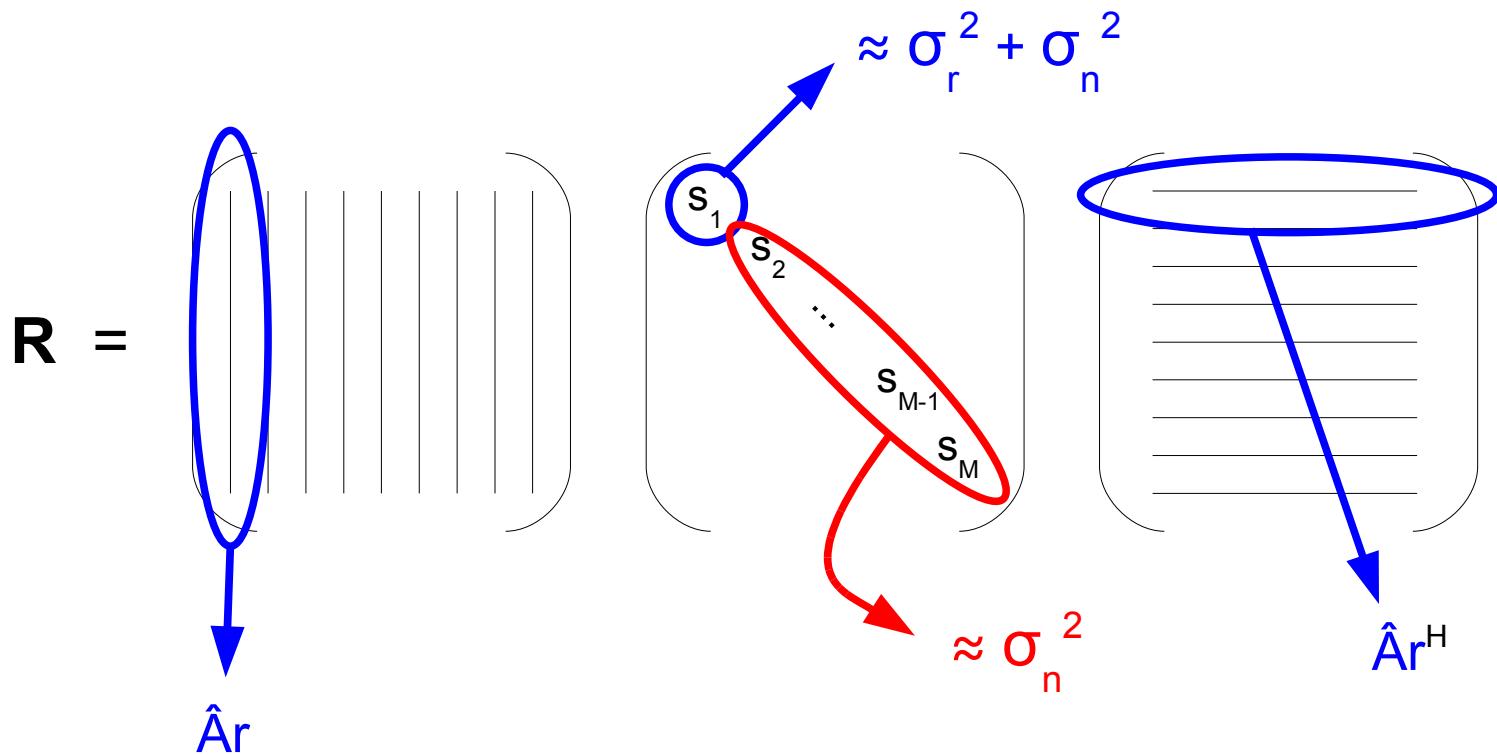
$$\mathbf{R} = \underbrace{\sigma_s^2 \cdot \mathbf{A}_s \mathbf{A}_s^H}_{\mathbf{R}_{\text{SOI}}} + \underbrace{\sigma_r^2 \cdot \mathbf{A}_r \mathbf{A}_r^H}_{\mathbf{R}_{\text{RFI}}} + \underbrace{\sigma_n^2 \cdot \mathbf{I}}_{\mathbf{R}_{\text{noise}}}$$

# RFI subspace estimation

Usual assumption:

$$P_{\text{noise}} \gg P_{\text{SOI}}$$

$$\mathbf{R} = \sigma_s^2 \cdot \mathbf{A}s \cdot \mathbf{A}s^H + \sigma_r^2 \cdot \mathbf{A}r \cdot \mathbf{A}r^H + \sigma_n^2 \cdot \mathbf{I}$$



# RFI subspace estimation

## « Time-lags » approach

$$\mathbf{R}(t, \tau) = \mathbf{R}_{\text{SOI}}(t, \tau) + \mathbf{R}_{\text{RFI}}(t, \tau) + \mathbf{R}_{\text{bruit}}(t, \tau)$$

For  $\tau > 0$  :

$$\underbrace{\mathbf{R}_{\text{SOI}}(t, \tau)}_{\rightarrow 0}$$

$$\underbrace{\mathbf{R}_{\text{bruit}}(t, \tau)}_{\rightarrow 0}$$

SOI and noise       $\rightarrow$  White, stationary and i.i.d processes  
 $\rightarrow r(t, \tau) = 0$  with  $\tau > 0$

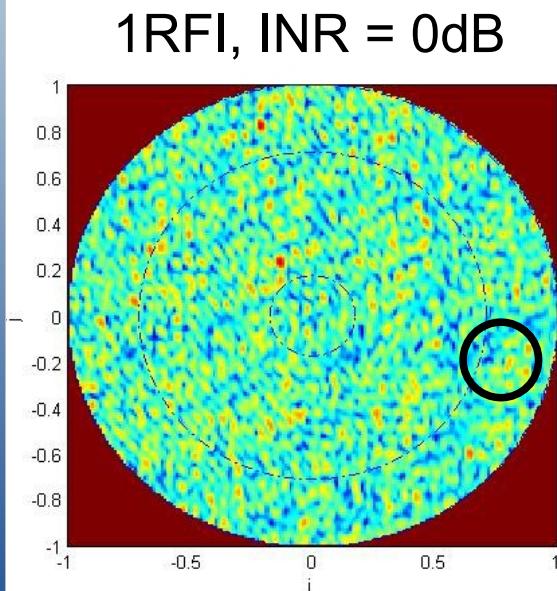
RFI                   $\rightarrow$  non-stationary process  
 $\rightarrow r(t, \tau_0) = \sigma_r^2(\tau_0)$  for any  $\tau_0$

For  $\tau_0 > 0$ :

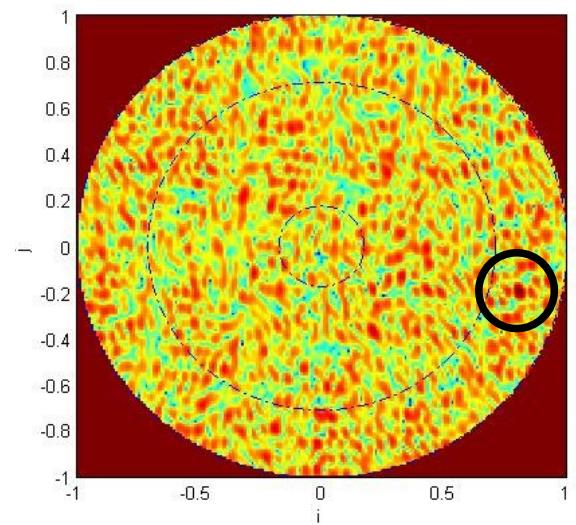
$$\mathbf{R}(t, \tau_0) = \mathbf{R}_{\text{RFI}}(t, \tau_0) = \sigma_r^2(\tau_0) \cdot \mathbf{A} \mathbf{r} \mathbf{A}^H$$

# RFI subspace estimation

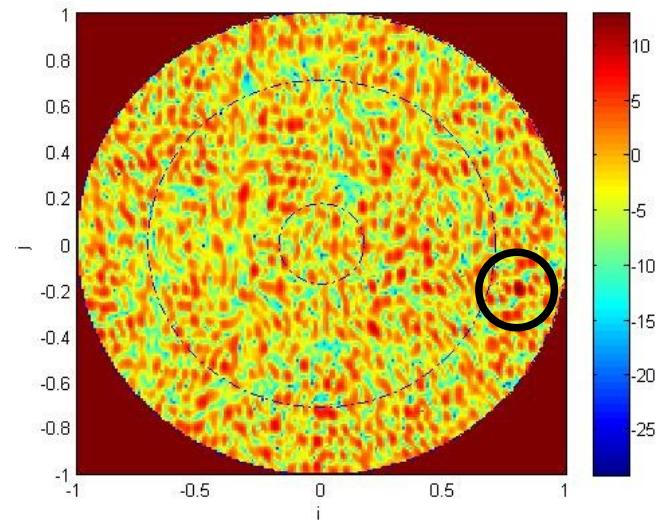
« Time-lags » approach



1 sample  
time-lag



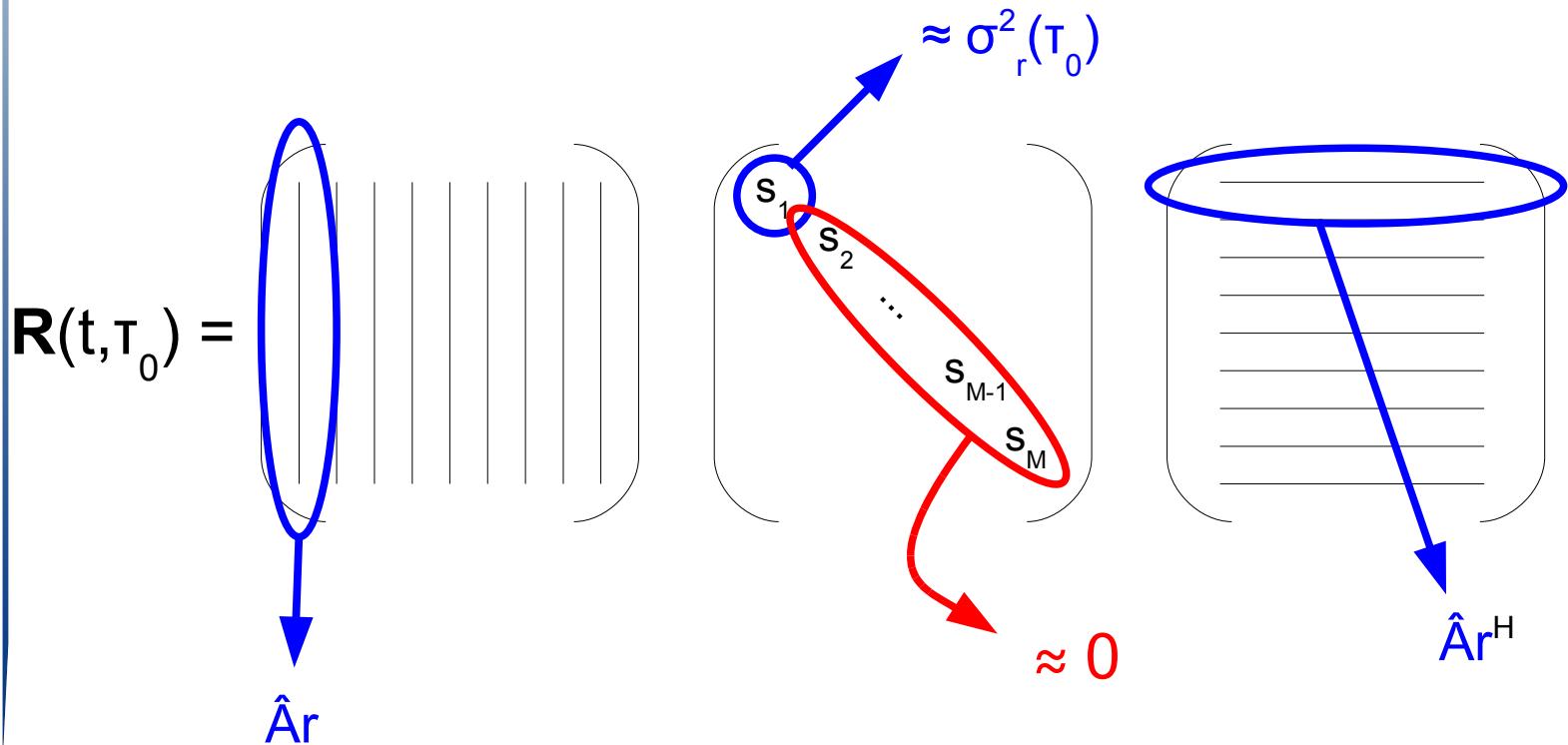
10 samples  
time-lag



# RFI subspace estimation

« Time-lags » approach

$$\mathbf{R}(t, \tau_0) = \underbrace{\mathbf{R}_{\text{sol}}(t, \tau_0) + \mathbf{R}_{\text{RFI}}(t, \tau_0)}_{\rightarrow 0} + \underbrace{\mathbf{R}_{\text{bruit}}(t, \tau_0)}_{\rightarrow 0} = \sigma_r^2(\tau_0) \cdot \mathbf{A} \mathbf{r}^H \cdot \mathbf{A}$$



# Implementation

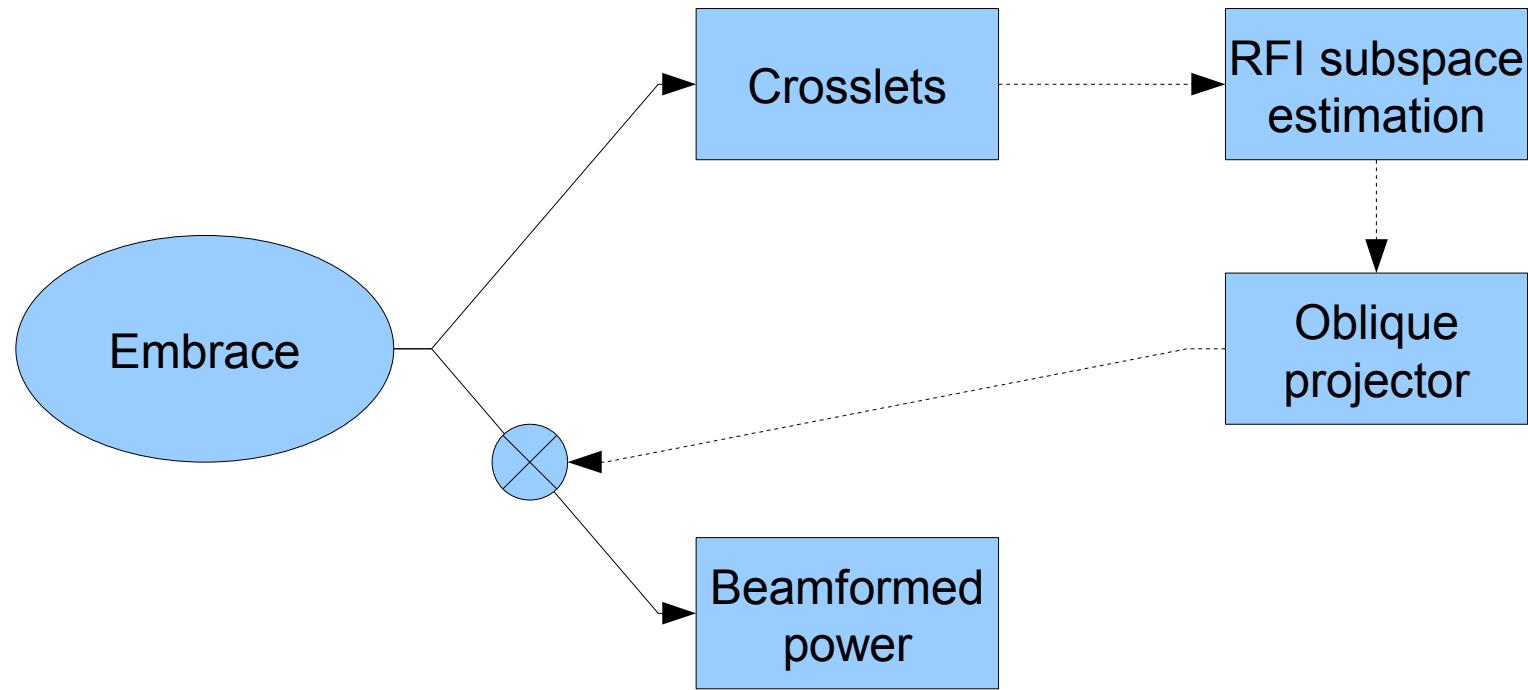
## Implementation : Embrace



Analog beamforming:  
16 tile sets pre-beamformed

- Frequency range :  
0.5-1.5 GHz
- Provides each second  
16x16 data covariance  
matrices (crosslets)
- 1 subband processed per  
second (per crosslet)
- full band processed within  
512s

# Implementation



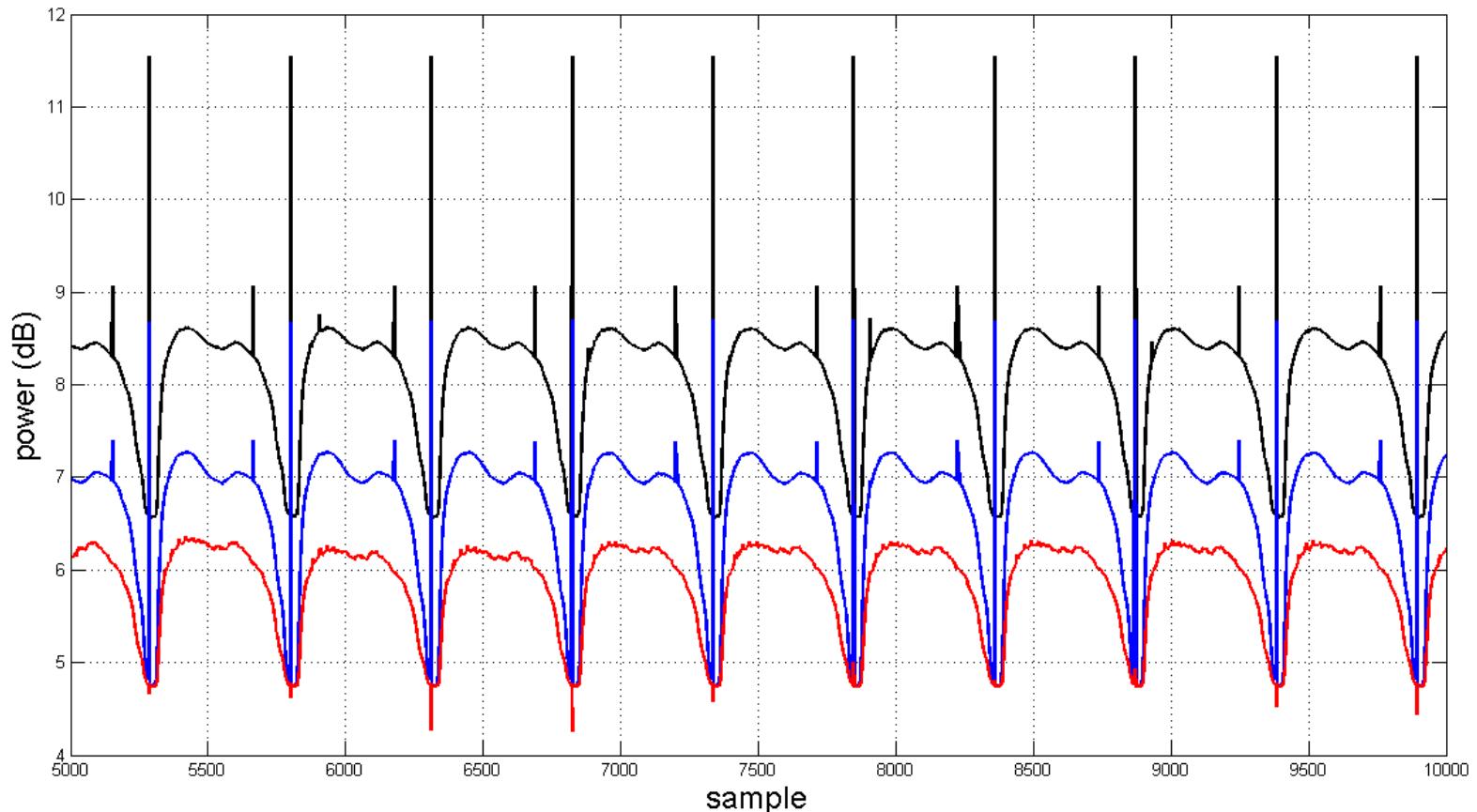
Single channel mode:

Steering vector updated each second  
Corrections applied with 1s delay

Full band mode:

Corrections applied with 512s delay!

# Implementation



25

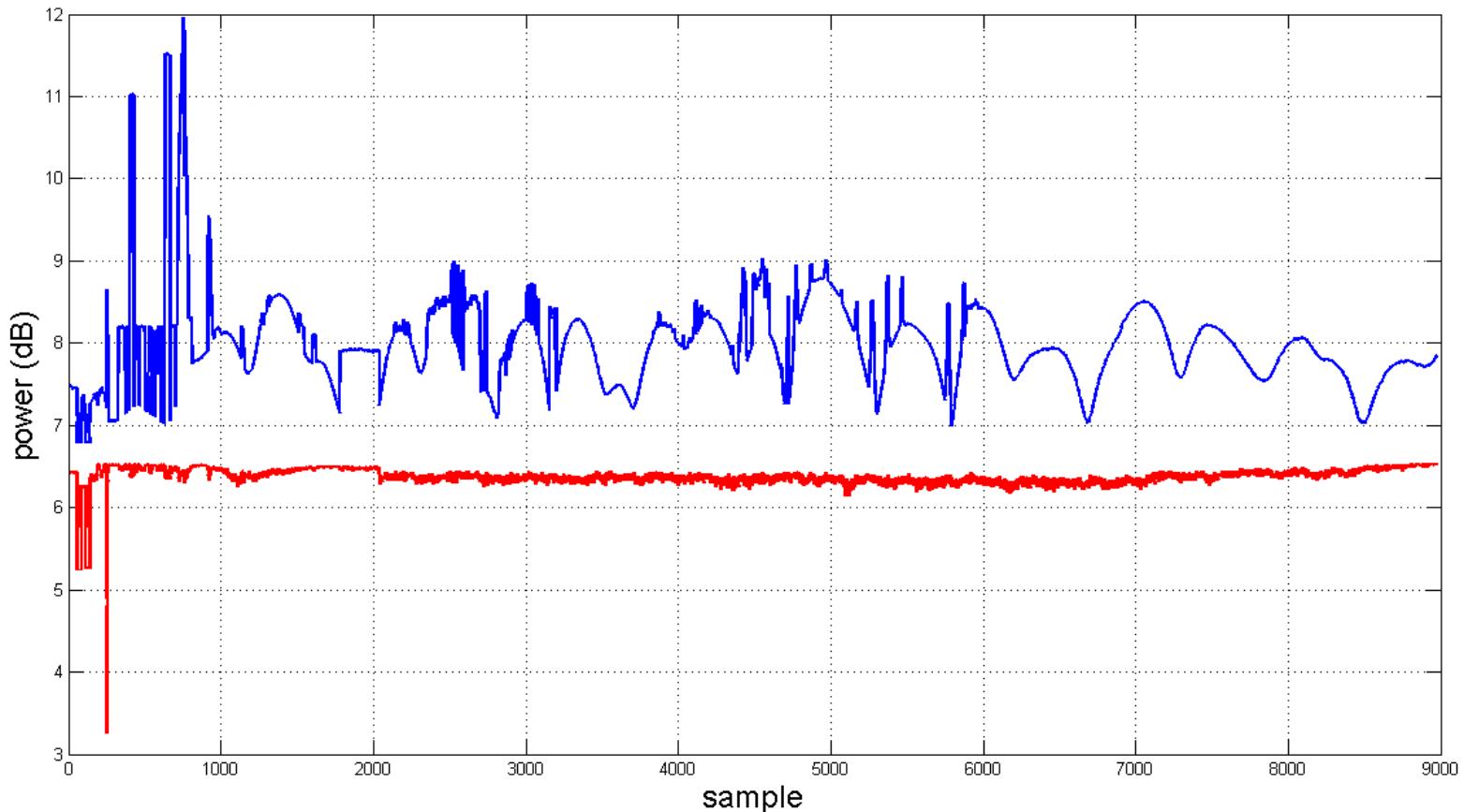
Total power

Digital beamforming  
(middle of the analog beam)

Processed data

# Implementation

## Single channel mode



Beamformed power (middle of the beam)

Data (offline) processed

# Conclusion

- Different efficient RFI mitigation algorithms exist
- Depending on the RT architecture, computational complexities can be improved
- End users are the first encountered limitation ("black box phobia"...)